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DEVELOPMENT AND EVALUATION OF SOFTWARE RELIABILITY ESTIMATORS,(U)

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1 OF 2

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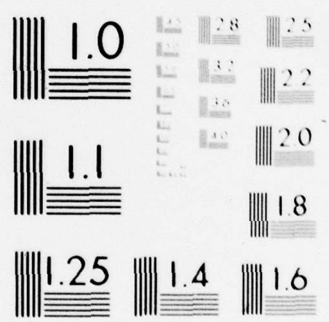

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SYSTEMS RESEARCH LABORATORY
DEPARTMENT OF ELECTRICAL ENGINEERING

UNIVERSITY OF UTAH
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UTEC SR 77-013

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DEVELOPMENT AND EVALUATION OF SOFTWARE RELIABILITY ESTIMATORS,

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1.0 Introduction

This study considers the problem of estimating the number of errors in a software package and its mean time-to-failure (MTTF). An emphasis is placed on the comparison and evaluation of various estimators and on determining the estimator accuracy. The estimation problem is described below.

1.1 The Estimation Problem

Consider a software package being tested to detect program errors. Let the testing start at t=0 and denote the error detection times by $t_1, t_2, \ldots t_n$. Also define the "inter-detection" times, x_i, the time intervals between the detection of errors, as

$$x_i = \begin{cases} t_i - t_{i-1} & i=2,3,\ldots n \\ t_i & i=1 \end{cases} \qquad (1\text{-}1)$$

The correction of errors can be done in two possible ways. The first possibility is to have all errors corrected immediately after detection. An equivalent method will be to correct the errors at any time after discovery but not to count rediscoveries of those errors as new ones. The second correction method accumulates the detected errors and at some discrete times, t_k, it corrects several errors n_k. The first method is called the Instant Correction method and it is discussed in Sections 2 and 3. The second method is called the Delayed Correction method and it is discussed in Sections 4 and 5.

It is assumed that the program size remains the same during the test phase. Furthermore, since there is no reliable information about the number of errors which are introduced during the corrections,

it is assumed that the number of the new errors is small and therefore, it is negligible. Then the objective is to estimate N, the initial number of errors in the program, and T, the MTTF after detecting n errors, from the sequence x_1, x_2, x_n.

First note that during the interval $t_{i-1} < t < t_i$ the number of errors in the program is constant. Therefore, it is reasonable to assume that the error detection rate will be constant too. The error detection rate, which is also called the hazard rate, is denoted by z_i. This assumption was made in all the reported models, except for the one by Schick and Wolverton [8], where it was assumed that the hazard increases linearly with time. Since we could not find a physical justification for this model in our case, we did not use it.

Once it is agreed that the hazard rate during $t_{i-1} < t < t_i$ is a constant and equals z_i, the probability density function for x_i can be derived [11]; it is found to be exponential.

$$f(x_i) = z_i e^{-z_i x_i} \tag{1-2}$$

The mean value of x_i, which is denoted by $\bar{T}_i$, is actually the MTTF before the detection of the i^{th} error. It is given by

$$\bar{T}_i = \frac{1}{z_i} \tag{1-3}$$

After some errors are corrected, the hazard function will vary. The two main models of Shooman [4] and Jelinski and Moranda [1] assume that the hazard function is proportional to the number of remaining errors. Therefore, the two models are equivalent for our case. We prefer to use the formulation of Jelinski as it gives $\hat{N}$

directly. The hazard function is therefore assumed to be:

$$z_i = \phi\ (N-i+1) \tag{1-4}$$

Where $(N-i+1)$ is the number of remaining errors during the time $t_{i-1} < t < t_i$ and ϕ is a positive constant. This model is called the Standard model.

A different relationship between z_i and i was suggested by Jelinski and Moranda. It is given by (1-5):

$$z_i = \lambda_o a^i \tag{1-5}$$

Both λ_o and a are positive constants. This model assumes that the hazard function changes by a constant ratio and therefore, it is called the Geometric model.

Another model was developed by Musa [7]. This model approximates the number of errors, which is an integer, by a continuous real number. Based on that, he found the expected number of errors to vary exponentially, and the mean value of t_i is given by

$$\bar{t}_i = \frac{-1}{\phi} \ln\ (1 - \frac{i}{N}) \tag{1-6}$$

This model is referred to as the exponential model.

The next step is to select the data to be used for estimation. While all the previous studies selected x_i as the data, it was found that the sequence t_i may give better results. The reason for this is that the t's are the integrals of the x's, and therefore will fluxuate less. We have used both the x_i and the t_i data for estimation with each estimator being designated as the x type or the t type.

Finally, when the model and the data are selected, one can still

select different types of estimators. The most common type is the Maximum-Likelihood (ML) one. This was used in all the reported studies. Another possible approach is to select the model parameters, N and ϕ, for example, in such a way that the difference between the data points and their mean values is minimized in a least square sense. For example, if we have x_i data, and we are using the standard model, we can find the mean value of x_i, which is denoted by $\bar{T}_i$, from (1-3) and (1-4).

$$\bar{T}_i = \frac{1}{\phi(N-i+1)} \tag{1-7}$$

and defines the estimation error E by (1-8)

$$E = \sum_{i=1}^{n} (x_i - \bar{T}_i)^2 = \sum_{i=1}^{n} \left(x_i - \frac{1}{\phi(N-i+1)}\right)^2 \tag{1-8}$$

This estimation method selects N and ϕ which minimize E. This type of estimator is called a least-square (LS) estimator. Note that it can be used with x or t type, as well as with the Standard, Geometric or the Exponential models.

Next we may select any combination of models, data type and estimation methods. However, some combinations lead to complex analyses; therefore, they are not used. The seven combinations which were selected are illustrated in Fig. 1-1. The resulting estimators for the Instant Correction methods are described in Section 2 and their results are evaluated in Section 3. Similar estimators were developed for the delayed correction method. It was found that the exponential model cannot be modified for this case and therefore, it was not used here. The other six estimators are described in

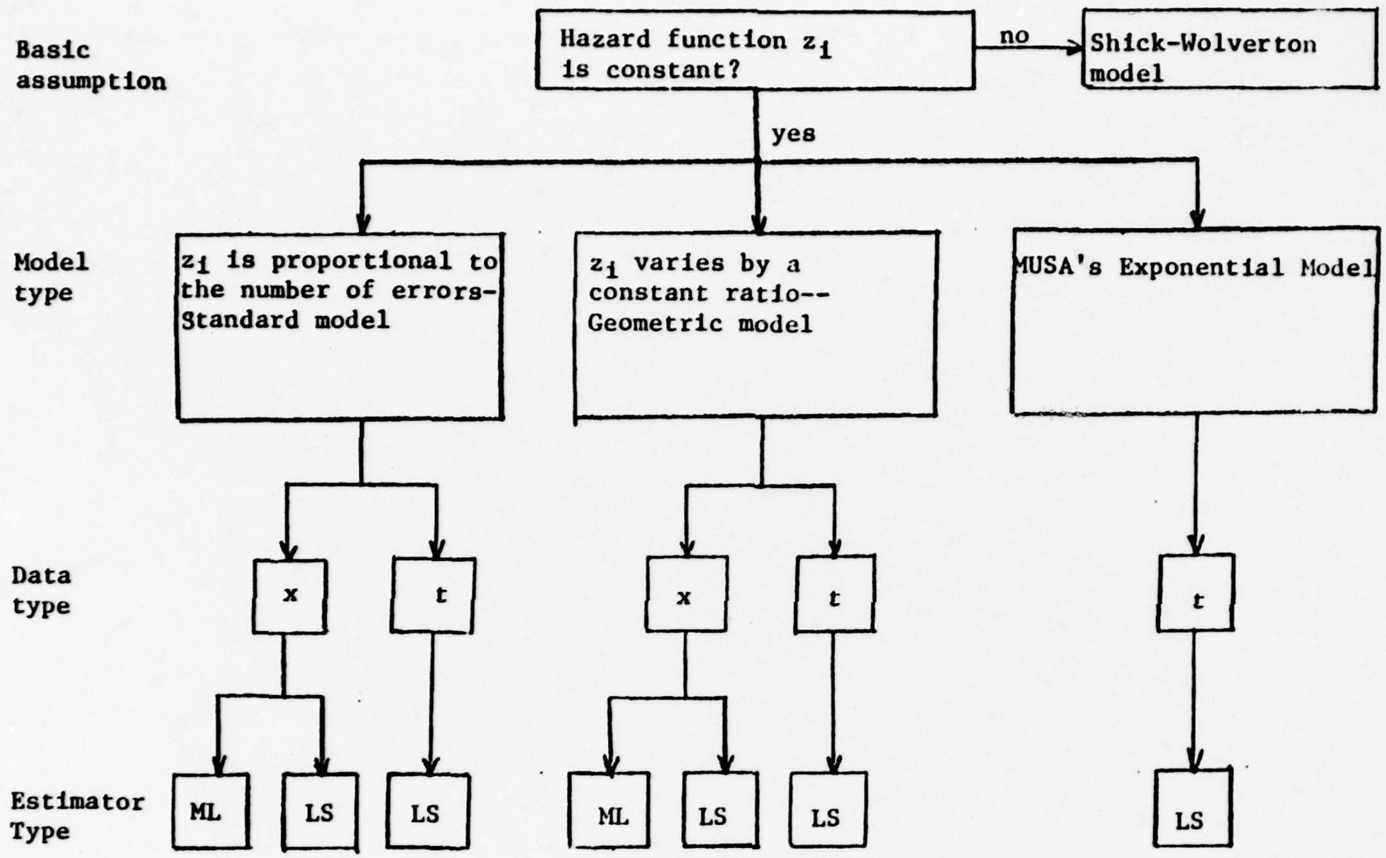

Fig. 1-1. The estimators and their relationship.

Section 4 and their results are evaluated in Section 5.

In addition to determining the estimates, we wish to learn more about their accuracy. This is done in two methods. The first method is the development of confidence intervals for the estimates. The second method is to study the effects of N and n on the accuracy of the estimate. The two methods, along with some experimental results are given in Section 6.

Finally, the format for the data collection is important as it describes the information to be gathered. This is especially important in the case of delayed error correction, where the detected errors should be grouped into several intervals. Furthermore, the source of each error should be discovered, so that errors would not be counted more than once, if the reliability models of Section 2 are to be used. In order to make sure that all the required data are collected, a proposed format was devised for data gathering. This is described in Appendix E.

2.0 Reliability Model and Estimators

Several methods are suggested for the estimation of reliability parameters. These are listed below along with their equations.

2.1 Maximum-Likelihood Model

Here it is assumed that the initial number of errors is N. Errors are detected at random and are corrected immediately. After correcting the $(i-1)^{th}$ error, the number of remaining errors in the program is $(N-i+1)$ and the hazard rate is assumed to be proportional to the number of the remaining errors. Thus, the hazard rate before detecting the i^{th} error is

$$z_i = \phi(N-i+1) \tag{2-1}$$

Assuming that n errors were detected and that the detection times are $t_1, t_2 \ldots t_n$. Define the time differences as

$$\begin{aligned} x_i &= t_i - t_{i-1} \qquad i = 2,3 \ldots n \\ x_1 &= t_1 \end{aligned} \tag{2-2}$$

Then it is possible to estimate N and ϕ by maximum likelihood estimators $\hat{N}$ and $\hat{\phi}$. The derivation, given in Appendix A.1 shows that $\hat{N}$ is the solution of

$$\sum_{i=1}^{n} \frac{1}{\hat{N}-i+1} - \frac{n}{\hat{N} - \dfrac{\sum^{n} (i-1)\, x_i}{\sum_{i=1}^{n} x_i}} = 0 \tag{2-3}$$

and

$$\hat{\phi} = \frac{\sum_{i=1}^{n} \dfrac{1}{\hat{N}-i+1}}{\sum_{i=1}^{n} x_i} \tag{2-4}$$

The MTTF after the $(i-1)^{th}$ error correction is

$$\hat{T}_i = \frac{1}{\hat{\phi}(\hat{N}-i+1)} \tag{2-5}$$

2.2 Geometric Maximum Likelihood Model

In this model, the hazard rate decreases by a constant ratio after the correction of an error. Accordingly, the hazard function before the detection of the i^{th} error is

$$z_i = \lambda_o a^i \tag{2-6}$$

where λ_o and a are constants.

Here one can estimate the most likely values of λ_o, a and the mean time to failure. This is done in Appendix A·2. The result of the analysis shows that the most likely estimtes are the solutions of (2-7) and (2-8)

$$\frac{n(n+1)}{2a} - \frac{n \sum_{i=1}^{n} i\, a^{i-1} x_i}{\sum_{i=1}^{n} a^i x_i} = 0 \tag{2-7}$$

$$\lambda_o = \frac{n}{\sum_{i=1}^{n} a^i x_i} \tag{2-8}$$

The mean time to failure after the correction of the $(i-1)^{th}$ error is

$$\hat{T}_i = \frac{1}{z_i} = \frac{1}{\lambda_o a^i} \tag{2-9}$$

2.3 Least Square x Model

The basic assumptions of this model are the same as those of the model in Section 2.1. However, instead of determining the most likely estimates of N and ϕ, we search for those values which minimize the sum of the squares of the deviations of x_i from the mean values.

$$E = \sum_{i=1}^{n} (x_i - \bar{x}_i)^2 = \sum_{i=1}^{n} [x_i - \frac{1}{\phi(N-i+1)}]^2 \tag{2-10}$$

N and ϕ which minimize E, as given by (2-10), are derived in A.3. It is found that $\hat{N}$ is the solution of the equation (2-11).

$$\sum_{i=1}^{n} \frac{x_i}{(N-i+1)^2} \cdot \sum_{i=1}^{n} \frac{1}{(N-i+1)^2} - \sum_{i=1}^{n} \frac{x_i}{(N-i+1)} \cdot \sum_{i=1}^{n} \frac{1}{(N-i+1)^3} = 0 \tag{2-11}$$

The estimate for ϕ is

$$\hat{\phi} = \frac{\sum_{i=1}^{n} \frac{1}{(N-i+1)^2}}{\sum_{i=1}^{n} \frac{x_i}{N-i+1}} \tag{2-12}$$

The MTTF, after the $(i-1)^{th}$ error correction is

$$\hat{T}_i = \frac{1}{z_i} = \frac{1}{\hat{\phi}(\hat{N}-i+1)} \tag{2-13}$$

2.4 Least Square t Model

This model differs from the previous one by the fact that we operate on the t times rather than the x times. Note that the times t_i are given by

$$t_i = \sum_{j=1}^{i} x_j \qquad (2\text{-}14)$$

The rationale for the selection of t_i as a quantity to fit is that it may be less sensitive to random changes, due to the summation of x_i. We compare t_i to its expected value, $\bar{t}_i$, which is given by

$$\bar{t}_i = \sum_{j=1}^{i} \bar{x}_j = \sum_{j=1}^{i} \frac{1}{(N-j+1)\phi} \qquad (2\text{-}15)$$

Consequently, the sum of the squares of the deviations is

$$E = \sum_{i=1}^{n} (t_i - \bar{t}_i)^2 = \sum_{i=1}^{n} \Big(t_i - \sum_{j=1}^{i} \frac{1}{\phi(N-j+1)}\Big)^2 \qquad (2\text{-}16)$$

The objective is to determine $\hat{N}$ and $\hat{\phi}$ which minimize (2-16). This is done in Appendix A·4 where it is shown that $\hat{N}$ is the solution of (2-17).

$$\sum_{i=1}^{n} t_i B_i \sum_{i=1}^{n} A_i^2 - \sum_{i=1}^{n} t_i A_i \sum_{i=1}^{n} A_i B_i = 0 \qquad (2\text{-}17a)$$

where

$$A_i = \sum_{j=1}^{i} \frac{1}{N-j+1} \qquad (2\text{-}17b)$$

and

$$B_i = \sum_{j=1}^{i} \frac{1}{(N-j+1)^2} \quad \text{(2-17c)}$$

$\hat{\phi}$, is determined from

$$\hat{\phi} = \frac{\sum_{i=1}^{n} A_i^2}{\sum_{i=1}^{n} t_i A_i} \quad \text{(2-18)}$$

The mean time to failure, $\hat{T}$, is given by

$$\hat{T}_i = \frac{1}{\hat{\phi}(\hat{N}-i+1)} \quad \text{(2-19)}$$

2.5 Geometric Least Square x Model

Consider the geometric model where the hazard function after the correction of the $(i-1)^{th}$ error is

$$z_i = \lambda_o a^i \quad \text{(2-20)}$$

The mean time between the detection of the $(i-1)^{th}$ and the i^{th} error is

$$\bar{x}_i = \frac{1}{z_i} = \frac{1}{\lambda_o a^i} \quad \text{(2-21)}$$

The objective now is to select the parameters a and λ_o such that the sum of the squares of deviations $(x_i - \bar{x}_i)$ is minimized. Thus, the estimation error is

$$E = \sum_{i=1}^{n} (x_i - \bar{x}_i)^2 = \sum_{i=1}^{n} (x_i - \frac{1}{\lambda_o a^i})^2 \quad \text{(2-22)}$$

The parameters $\hat{\lambda}_o$ and $\hat{a}$ which minimize E are derived in Section A.5. It is found that a is the solution of (2-23), and $\hat{\lambda}_o$ is obtained from (2-24).

$$\sum_{i=1}^{n} \frac{i\, x_i}{a^i} \sum_{i=1}^{n} \frac{1}{a^{2i}} - \sum_{i=1}^{n} \frac{x_i}{a^i} \sum_{i=1}^{n} \frac{i}{a^{2i}} = 0 \tag{2-23}$$

$$\lambda_o = \frac{\sum_{i=1}^{n} \frac{1}{a^{2i}}}{\sum_{i=1}^{n} \frac{x_i}{a^i}} \tag{2-24}$$

The mean time to failure for the i^{th} error, $\hat{T}_i$, is given by

$$\hat{T}_i = \frac{1}{\hat{\lambda}_o \hat{a}^i} \tag{2-25}$$

2.6 Geometric Least Square t Model

The geometric least square model is applied to the cumulative time to failure t_i. Consequently, the estimation error, E, is

$$E = \sum_{i=1}^{n} (t_i - \bar{t}_i)^2 \tag{2-26}$$

where

$$t_i = \sum_{j=1}^{i} x_j \tag{2-27}$$

and

$$\bar{t}_i = \sum_{j=1}^{i} \bar{x}_j = \sum_{j=1}^{i} \frac{1}{\lambda_o a^j} \tag{2-28}$$

Therefore, E may be written as

$$E = \sum_{i=1}^{n} \left(t_i - \sum_{j=1}^{i} \frac{1}{\lambda_o a^j}\right)^2 \tag{2-29}$$

The parameters, $\hat{a}$ and $\hat{\lambda}_o$ which minimize E are derived in Section A.6. $\hat{a}$ is the solution of eq. (2-30)

$$\sum_{i=1}^{n} t_i D_i \sum_{i=1}^{n} C_i^2 - \sum_{i=1}^{n} t_i C_i \sum_{i=1}^{n} C_i D_i = 0 \tag{2-30a}$$

where

$$C_i = \sum_{j=1}^{i} \frac{1}{a^j} \tag{2-30b}$$

and

$$D_i = \sum_{j=1}^{i} \frac{j}{a^j} \tag{2-30c}$$

$\hat{\lambda}_o$ is found from:

$$\lambda_o = \frac{\sum_{i=1}^{n} C_i^2}{\sum_{i=1}^{n} t_i C_i} \tag{2-31}$$

and $\hat{T}_i$, is given by

$$\hat{T}_i = \frac{1}{\hat{\lambda}_o \hat{a}^i} \tag{2-32}$$

2.7 Exponential Least Square Model

The following model is based on the work reported by Musa [7], where a continuous model is assumed for N. According to that model, the expected number for the corrected errors, N_c, is

$$N_c = N(1-e^{-\phi t}) \tag{2-33}$$

where ϕ is a constant and N is the initial number of the errors. Also the expected time until the i^{th} error is detected, t_i, is found to be

$$\bar{t}_i = \frac{-1}{\phi} \ln \left(1 - \frac{i}{N}\right) \tag{2-34}$$

Accordingly, we seek the parameters $\hat{\phi}$ and $\hat{N}$ which minimize the estimation error E.

$$E = \sum_{i=1}^{n} (t_i - \bar{t}_i)^2 = \sum_{i=1}^{n} \left[t_i + \frac{1}{\phi} \ln \left(1 - \frac{i}{N}\right)\right]^2 \tag{2-35}$$

It is shown in Appendix A that the best estimate for N is the solution of (2-36)

$$\sum_{i=1}^{n} \frac{i\, t_i}{N-i} \sum_{i=1}^{n} \ln^2 \left(\frac{N-i}{N}\right) - \sum_{i=1}^{n} t_i \ln \left(\frac{N-i}{N}\right) \sum_{i=1}^{n} \frac{i}{N-i} \ln \left(\frac{N-i}{N}\right) = 0 \tag{2-36}$$

The estimate for ϕ is found from

$$\phi = \frac{\sum_{i=1}^{n} \ln^2 \left(\frac{N}{N-i}\right)}{\sum_{i=1}^{n} t_i \ln \left(\frac{N}{N-i}\right)} \tag{2-37}$$

The MTTF before the i^{th} error, $\hat{T}_i$, is given by

$$\hat{T}_i = \bar{t}_i - \bar{t}_{i-1} \tag{2-38}$$

where $\bar{t}_i$ is given by (2-34). This is found to be

$$\hat{T}_i = \frac{1}{\hat{\phi}} \ln \frac{\hat{N}-i+1}{\hat{N}-1} \tag{2-39}$$

3.0 Test and Evaluation of Estimators

The various estimators which are described in Section 2 were tested and evaluated in order to verify that the equations are correct and have no errors. Also, we want to determine the quality of the estimators from the points of view of convergence and accuracy.

The first task is relatively easy. It was done by testing the estimators with deterministic data rather than random. In other words, instead of having a sequence of random numbers, x_i, to analyze, we generate a sequence of the expected values of x_i for the parameters $N=60$, $n=50$, $\phi = 0.1$, with the corresponding MTTF of $T = 1.0$. Since the data is not random, all the estimates should estimate the parameters N and T exactly. This was finally achieved after correcting several errors in the program. This method was found very useful in debugging the estimators program.

The next task is more difficult as real data is not available for testing. The next best thing to real data is a randomly generated data with the desired exponential probability function. Thus, the data was generated randomly with exponential probability density function,

$$f(x_i) = \phi(N-i+1)\ e^{-\phi(N-i+1)x_i} \tag{3-1}$$

where the index i, was adjusted for each simulated time. Another point of significance in simulating the data was to verify that the generated time intervals, x_i, are independent of each other. This was examined by defining the variable y_i by (3-2).

$$y_i = e^{-\phi(N-i+1)x_i} \tag{3-2}$$

The resulting random variable, y_i, is uniformly distributed in the interval (0,1). In order to examine the dependence between the various y_i's, we evaluated the correlation function R(k).

$$R(k) = \frac{1}{n-k} \sum_{i=1}^{n-k} (y_i - \frac{1}{2})(y_{i+k} - \frac{1}{2}) \qquad (3\text{-}3)$$

It was found that R(0) equals 0.08, as expected, but all the quantities R (k), for k between 1 and 10 were close to zero, indicating that the y_i values, and therefore, the x_i values too, are independent.

In order to make the test statistically significant, 1000 random sequences were generated for each estimator and the parameters were estimated. The test results are presented as histograms which show the frequency of estimating a certain parameter.

Following are four histograms for estimating N, the initial number of errors. The data was generated randomly on the basis of the parameters N=60 and ϕ=0.02; the right estimate for N will be 60. However, due to the randomness of the data, the estimates are spread over a wide range. The histograms indicate the frequency of estimating $\hat{N}$. Also included is the cumulative frequency (C.D.F.), which indicates the number of estimates being less than or equal to a certain value of $\hat{N}$. Note that all the histograms are similar in shape, indicating that all the four estimators are similar to their behavior.

Comparing the histograms we observe that their general shape is the same. The median point, for which 50% of the estimates fall below it, is 60 and 61 for the estimates. The convergence rate is very high in all the four models. It varied between 996 and 999 converging samples, out of 1000.

HISTOGRAM OF NO EST

USING JELINSKI MAXIMUM LIKELIHOOD MODEL

DATA PARAMETERS - N#50 . NO# 60 . PHI#0.100E 00 . SAMPLES#1000

NO EST	FREQ	C.D.F.	PLOT
51	6	6	.XXXXXX
52	32	38	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
53	42	80	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
54	57	137	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
55	68	205	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
56	70	275	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
57	65	340	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
58	63	403	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
59	69	472	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
60	43	515	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
61	49	564	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
62	39	603	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
63	37	640	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
64	34	674	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
65	32	706	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
66	23	729	.XXXXXXXXXXXXXXXXXXXXXXX
67	25	754	.XXXXXXXXXXXXXXXXXXXXXXXXX
68	19	773	.XXXXXXXXXXXXXXXXXXX
69	24	797	.XXXXXXXXXXXXXXXXXXXXXXXX
70	18	815	.XXXXXXXXXXXXXXXXXX
71	23	838	.XXXXXXXXXXXXXXXXXXXXXXX
72	11	849	.XXXXXXXXXXX
73	8	857	.XXXXXXXX
74	13	870	.XXXXXXXXXXXXX
75	12	882	.XXXXXXXXXXXX
76	8	890	.XXXXXXXX
77	12	902	.XXXXXXXXXXXX
78	7	909	.XXXXXXX
79	7	916	.XXXXXXX
80	5	921	.XXXXX
81	5	926	.XXXXX
82	3	929	.XXX
83	6	935	.XXXXXX
84	3	939	.XXX
85	6	944	.XXXXXX
86	2	946	.XX
87	3	949	.XXX
88	2	951	.XX
89	5	956	.XXXXX
91	1	957	.X
92	1	958	.X
93	1	959	.X
94	1	960	.X
95	1	961	.X
96	6	967	.XXXXXX
98	1	968	.X
99	1	969	.X
100	1	970	.X
102	2	972	.XX
103	3	975	.XXX
104	1	976	.X
106	1	977	.X
107	2	979	.XX
110	1	980	.X
111	1	981	.X
113	2	983	.XX
115	2	985	.XX
117	2	987	.XX
122	1	988	.X
123	1	989	.X
139	1	990	.X
141	1	991	.X
146	1	992	.X
160	1	993	.X
173	1	994	.X
199	1	995	.X
205	1	996	.X
220	1	997	.X
417	1	998	.X
937	1	999	.X

	1	1000	DID NOT CONVERGE

HISTOGRAM OF NC EST

USING LEAST SQUARES FIT OF JELINSKI MODEL TO THE XBS

DATA PARAMETERS - N=50 , NO= 60 , PHI=0.100E 00 , SAMPLES=1000

NO EST	FREQ	C.D.F.	PLOT
---	26	26	DID NOT CONVERGE
50	20	46	.XXXXXXXXXXXXXXXXXXXX
51	27	73	.XXXXXXXXXXXXXXXXXXXXXXXXXXX
52	41	114	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
53	39	153	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
54	35	188	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
55	39	227	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
56	51	278	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
57	39	317	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
58	58	375	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
59	35	410	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
60	45	455	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
61	52	507	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
62	44	551	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
63	30	581	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
64	37	618	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
65	26	644	.XXXXXXXXXXXXXXXXXXXXXXXXXX
66	29	673	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXX
67	22	695	.XXXXXXXXXXXXXXXXXXXXXX
68	18	713	.XXXXXXXXXXXXXXXXXX
69	18	731	.XXXXXXXXXXXXXXXXXX
70	17	748	.XXXXXXXXXXXXXXXXX
71	22	770	.XXXXXXXXXXXXXXXXXXXXXX
72	16	786	.XXXXXXXXXXXXXXXX
73	19	805	.XXXXXXXXXXXXXXXXXXX
74	7	812	.XXXXXXX
75	11	823	.XXXXXXXXXXX
76	16	839	.XXXXXXXXXXXXXXXX
77	6	845	.XXXXXX
78	18	863	.XXXXXXXXXXXXXXXXXX
79	12	875	.XXXXXXXXXXXX
80	8	883	.XXXXXXXX
81	4	887	.XXXX
82	4	891	.XXXX
83	11	902	.XXXXXXXXXXX
84	9	911	.XXXXXXXXX
85	13	924	.XXXXXXXXXXXXX
86	8	932	.XXXXXXXX
87	2	934	.XX
88	3	937	.XXX
89	4	941	.XXXX
90	3	944	.XXX
91	2	946	.XX
92	4	950	.XXXX
95	2	952	.XX
96	3	955	.XXX
97	1	956	.X
99	1	957	.X
100	1	958	.X
101	1	959	.X
102	1	960	.X
103	2	962	.XX
104	1	963	.X
105	1	964	.X
106	1	965	.X
107	3	968	.XXX
108	1	969	.X
109	1	970	.X
112	1	971	.X
113	2	973	.XX
114	1	974	.X
115	1	975	.X
120	2	977	.XX
124	1	978	.X
127	3	981	.XXX
129	3	984	.XXX
130	1	985	.X
131	2	987	.XX
135	2	989	.XX
143	1	990	.X
144	1	991	.X
161	1	992	.X
163	1	993	.X
165	1	994	.X
184	1	995	.X
196	1	996	.X
243	1	997	.X
541	1	998	.X
---	2	1000	DID NOT CONVERGE

HISTOGRAM OF NO EST

USING LEAST SQUARES FIT OF JELINSKI MODEL TO THE TBS

DATA PARAMETERS - N=50 , NO= 63 , PHI=0.100E 00 , SAMPLES=1000

NO EST	FREQ	C.FREQ	PLOT
~~50~~	~~1~~	~~1~~	~~.X~~
51	10	11	.XXXXXXXXXX
~~52~~	~~33~~	~~43~~	~~.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX~~
53	57	100	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
~~54~~	~~63~~	~~163~~	~~.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX~~
55	82	230	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
~~56~~	~~77~~	~~316~~	~~.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX~~
57	62	370	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
~~58~~	~~60~~	~~430~~	~~.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX~~
59	48	478	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
~~60~~	~~46~~	~~524~~	~~.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX~~
61	51	575	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
~~62~~	~~38~~	~~613~~	~~.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX~~
63	46	659	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
~~64~~	~~18~~	~~677~~	~~.XXXXXXXXXXXXXXXXXX~~
65	31	708	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
~~66~~	~~26~~	~~734~~	~~.XXXXXXXXXXXXXXXXXXXXXXXXXX~~
67	19	753	.XXXXXXXXXXXXXXXXXXX
~~68~~	~~27~~	~~780~~	~~.XXXXXXXXXXXXXXXXXXXXXXXXXXX~~
69	15	795	.XXXXXXXXXXXXXXX
~~70~~	~~17~~	~~812~~	~~.XXXXXXXXXXXXXXXXX~~
71	13	825	.XXXXXXXXXXXXX
~~72~~	~~9~~	~~834~~	~~.XXXXXXXXX~~
73	10	844	.XXXXXXXXXX
~~74~~	~~11~~	~~855~~	~~.XXXXXXXXXXX~~
75	6	861	.XXXXXX
~~76~~	~~13~~	~~874~~	~~.XXXXXXXXXXXXX~~
77	6	880	.XXXXXX
~~78~~	~~9~~	~~889~~	~~.XXXXXXXXX~~
79	9	898	.XXXXXXXXX
~~80~~	~~5~~	~~903~~	~~.XXXXX~~
81	1	904	.X
~~82~~	~~5~~	~~909~~	~~.XXXXX~~
83	3	912	.XXX
~~84~~	~~5~~	~~917~~	~~.XXXXX~~
85	1	918	.X
~~86~~	~~3~~	~~921~~	~~.XXX~~
87	6	927	.XXXXXX
~~88~~	~~5~~	~~932~~	~~.XXXXX~~
89	5	937	.XXXXX
~~90~~	~~2~~	~~939~~	~~.XX~~
91	5	944	.XXXXX
~~92~~	~~1~~	~~945~~	~~.X~~
93	1	946	.X
~~94~~	~~4~~	~~950~~	~~.XXXX~~
95	1	951	.X
~~96~~	~~2~~	~~953~~	~~.XX~~
99	2	955	.XX
~~100~~	~~1~~	~~956~~	~~.X~~
101	2	958	.XX
~~102~~	~~3~~	~~961~~	~~.XXX~~
103	1	962	.X
104	1	963	.X
105	1	964	.X
~~[illegible]~~	~~3~~	~~967~~	~~.XXX~~
113	2	969	.XX
~~[illegible]~~	~~2~~	~~971~~	~~.XX~~
117	1	972	.X
~~[illegible]~~	~~1~~	~~973~~	~~.X~~
120	1	974	.X
~~121~~	~~2~~	~~976~~	~~.XX~~
127	1	977	.X
~~[illegible]~~	~~2~~	~~979~~	~~.XX~~
132	1	980	.X
~~[illegible]~~	~~1~~	~~981~~	~~.X~~
137	1	982	.X
~~141~~	~~2~~	~~984~~	~~.XX~~
145	2	986	.XX
~~[illegible]~~	~~1~~	~~987~~	~~.X~~
149	1	988	.X
~~[illegible]~~	~~1~~	~~989~~	~~.X~~
174	1	990	.X
~~[illegible]~~	~~1~~	~~991~~	~~.X~~
206	1	992	.X
~~[illegible]~~	~~1~~	~~993~~	~~.X~~
212	1	994	.X
~~[illegible]~~	~~1~~	~~995~~	~~.X~~
819	1	996	.X

~~4 1000 DID NOT CONVERGE~~

HISTOGRAM OF NO EST

USING LEAST SQUARES FIT OF MUSA MODEL TO THE TBS

DATA PARAMETERS - N=50 , NO= 63 , PM(=0.100E 00 , SAMPLES=1000

NO EST	FREQ	C.D.F.	PLOT
---	1	1	DID NOT CONVERGE
51	2	3	.XX
~~52~~	~~[illegible]~~	~~30~~	[illegible]
53	39	68	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
~~54~~	~~[illegible]~~	~~119~~	[illegible]
55	68	187	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
~~56~~	~~[illegible]~~	~~276~~	[illegible]
57	74	350	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
~~58~~	~~46~~	~~396~~	[illegible]
59	53	449	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
~~60~~	~~56~~	~~505~~	[illegible]
61	44	549	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
~~62~~	~~44~~	~~593~~	[illegible]
63	47	646	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
~~64~~	~~[illegible]~~	~~663~~	[illegible]
65	22	691	.XXXXXXXXXXXXXXXXXXXXXX
~~66~~	~~33~~	~~724~~	[illegible]
67	19	743	.XXXXXXXXXXXXXXXXXXX
~~68~~	~~25~~	~~768~~	[illegible]
69	18	786	.XXXXXXXXXXXXXXXXXX
~~70~~	~~20~~	~~806~~	[illegible]
71	12	810	.XXXXXXXXXXXX
~~72~~	~~8~~	~~826~~	[illegible]
73	14	849	.XXXXXXXXXXXXXX
~~74~~	~~11~~	~~851~~	[illegible]
75	6	857	.XXXXXX
~~76~~	~~14~~	~~871~~	[illegible]
77	5	876	.XXXXX
~~78~~	~~9~~	~~885~~	[illegible]
79	3	893	.XXXXXXXX
~~80~~	~~5~~	~~899~~	~~.XXXXX~~
81	6	904	.XXXXXX
~~82~~	~~1~~	~~905~~	~~.X~~
83	5	910	.XXXXX
~~84~~	~~4~~	~~914~~	~~.XXXX~~
85	3	917	.XXX
~~86~~	~~2~~	~~919~~	~~.XX~~
87	3	922	.XXX
~~88~~	~~9~~	~~931~~	[illegible]
89	4	935	.XXXX
~~90~~	~~3~~	~~938~~	~~.XXX~~
91	2	940	.XX
~~92~~	~~4~~	~~944~~	~~.XXXX~~
93	2	946	.XX
~~94~~	~~1~~	~~947~~	~~.X~~
95	3	950	.XXX
~~96~~	~~1~~	~~951~~	~~.X~~
97	2	953	.XX
~~99~~	~~1~~	~~954~~	~~.X~~
100	2	956	.XX
101	2	958	.XX
103	3	961	.XXX
~~104~~	~~2~~	~~963~~	~~.XX~~
106	1	964	.X
~~108~~	~~3~~	~~967~~	~~.XXX~~
110	1	968	.X
~~111~~	~~1~~	~~969~~	~~.X~~
114	1	970	.X
~~115~~	~~1~~	~~971~~	~~.X~~
119	1	972	.X
~~120~~	~~2~~	~~974~~	~~.XX~~
122	2	976	.XX
~~127~~	~~1~~	~~977~~	~~.X~~
128	1	978	.X
~~129~~	~~1~~	~~979~~	~~.X~~
132	1	980	.X
~~133~~	~~1~~	~~981~~	~~.X~~
138	1	982	.X
~~141~~	~~2~~	~~984~~	~~.XX~~
146	2	986	.XX
~~147~~	~~1~~	~~987~~	~~.X~~
149	1	988	.X
~~160~~	~~1~~	~~989~~	~~.X~~
175	1	990	.X
~~[illegible]~~	~~1~~	~~991~~	~~.X~~
207	1	992	.X
~~211~~	~~1~~	~~993~~	~~.X~~
213	1	994	.X
~~[illegible]~~	~~1~~	~~995~~	~~.X~~
417	1	996	.X

---------+-----100%----DID NOT CONVERGE---------

Another feature of interest is the estimate of the mean time to failure, T. This parameter was estimated by all the seven methods. The estimators operated on random data which was generated for the parameters N=60, n=50, and ϕ = 0.1. The MTTF after detecting the 50 errors should be 1.0, however, the estimate varies due to the randomness of the data. A histogram for the geometric maximum likelihood model is given below. In this case all the estimates converged between the values of 0.25 and 1.75.

The convergence was not always so good especially in the models which estimated N first. A summary of the results is given in Table 3.1. The table contains the number of samples for which the estimate has converged. Also, it gives the values of $\hat{T}$, the MTTF, for various percentiles of the estimates. For example, the first row indicates that 25% of the estimates of $\hat{T}$, using the maximum likelihood method, were below 0.70. The results of Table 3.1 allow us to conclude the following:

a) The estimates of T which are based on the Geometric models are better. The variance of the estimate is smaller than that resulting from the standard or the exponential models. The reason for this is that the geometric model does not require the estimate of N, which is very sensitive to random variations in the data.

b) The least square estimates which are based on the x data are always worse than those based on the t data. The reason for this is that the t times are the summation of the x's, and therefore, they "smooth" out the randomness of x.

HISTOGRAM OF TTF EST

USING GEOMETRIC MAXIMUM LIKELIHOOD MODEL

DATA PARAMETERS - N=50 , N0= 60 , PHI=0.100E 00 , SAMPLES=1000

TTF EST	FREQ	C.D.F.	PLOT
0.250	1	1	.X
0.300	6	7	.XXXXXX
0.350	13	20	.XXXXXXXXXXXXX
0.400	33	53	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
0.450	48	101	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
0.500	77	178	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
0.550	81	259	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
0.600	83	342	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
0.650	99	441	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
0.700	116	557	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
0.750	89	646	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
0.800	82	728	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
0.850	58	786	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
0.900	63	849	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
0.950	44	893	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
1.000	24	917	.XXXXXXXXXXXXXXXXXXXXXXXX
1.050	26	943	.XXXXXXXXXXXXXXXXXXXXXXXXXX
1.100	12	955	.XXXXXXXXXXXX
1.150	18	973	.XXXXXXXXXXXXXXXXXX
1.200	9	982	.XXXXXXXXX
1.250	5	987	.XXXXX
1.300	5	992	.XXXXX
1.400	1	993	.X
1.450	1	994	.X
1.500	2	996	.XX
1.600	2	998	.XX
1.750	2	1000	.XX

Table 3.1 Summary of results of MTTF estimates

Method	No. of samples for which estimate Converged	Estimate Percentile					Estimate Range	
		10%	25%	50%	75%	90%	Min.	Max.
Maximum Likelihood	999	0.55	0.70	0.95	1.40	2.00	0.30	7.15
Least Square x Model	972	0.50	0.65	0.95	1.40	2.85	0.30	22.6
Least square t Model	996	0.50	0.70	1.00	1.45	2.20	0.25	21.1
Exponential	995	0.50	0.70	0.95	1.35	1.90	0.25	5.60
Geometric Max. Likelihood	1000	0.45	0.55	0.70	0.85	1.00	0.25	1.75
Geometric least-square x	953	0.45	0.60	0.75	1.05	1.90	0.05	6.75
Geometric least-square t	1000	0.45	0.55	0.70	0.85	1.00	0.20	1.70

c) We note that the general spread of the estimates of $\hat{T}$ is much smaller than that of $\hat{N}$. The reason is that $\hat{T}$ is derived from the detection rate, whereas $\hat{N}$ has to be found from the change in the error detection rate and a change is more sensitive to randomness, the way that the derivative function is more sensitive to noise.

Another question which interest us is the correlation between estimators. That is, if one estimator produces a large estimate by one method, would the same set of data produce large estimates using the other methods? In order to examine this, we have determined all the seven estimators for 20 sets of data. Note that although the data was generated randomly, the same sets of data were applied to all the estimators. The results were the estimates of $\hat{N}$ and $\hat{T}$, by the Standard and the Exponential models, and estimates for $\hat{a}$ and $\hat{T}$ by the Geometric models. All the data points were generated for the parameters N=60, n=50 and ϕ=0.1, with the resulting T=1.0. Thus, the correct values are N=60 and T=1.0. The actual estimates for all the 20 samples are given in Table 3.2.

An examination of the results of Table 3.2 reveal several interesting points:

a) There is a strong correlation between the estimators. When a set of data produces a small estimate of N, it does it with all the estimators (samples 1, 3, 7, 9, 15). Similarly, when the estimates are high, they are high with all the estimators, (samples 5, 10, 12, 13).

b) Whenever the estimate of N is high, the estimate of T is

Table 3.2 Comparison between estimators with identical data samples

N=60 n=50 ϕ=0.1 T=1.0

Sample Number	Maximum Likelihood		Least-Square x Model		Least-Square t Model		Exponential		Geometric Max. Lik.		Geometric L-S x Model		Geometric L-S t Model	
	$\hat{N}$	$\hat{T}$	$\hat{N}$	$\hat{T}$	$\hat{N}$	$\hat{T}$	$\hat{N}$	$\hat{T}$	$\hat{a}$	$\hat{T}$	$\hat{a}$	$\hat{T}$	$\hat{a}$	$\hat{T}$
1	55	1.21	55	1.22	56	1.15	56	1.07	0.957	0.73	0.950	0.82	0.960	0.65
2	62	0.73	63	0.71	62	0.74	63	0.70	0.971	0.55	0.966	0.59	0.970	0.54
3	56	1.39	56	1.36	55	1.57	55	1.45	0.966	0.71	0.945	1.02	0.960	0.77
4	63	0.70	57	0.97	73	0.50	73	0.50	0.974	0.51	0.960	0.64	0.979	0.43
5	69	0.57	81	0.47	63	0.73	63	0.71	0.974	0.50	0.977	0.48	0.969	0.59
6	60	0.79	57	0.98	61	0.72	62	0.68	0.974	0.49	0.946	0.81	0.975	0.45
7	54	1.80	58	1.26	53	2.31	54	1.95	0.956	0.92	0.948	1.11	0.953	0.99
8	63	0.96	66	0.86	61	1.05	62	0.99	0.971	0.74	0.968	0.76	0.969	0.77
9	55	1.61	55	1.54	56	1.35	57	1.22	0.953	0.98	0.958	1.00	0.961	0.80
10	66	0.75	68	0.70	65	0.77	65	0.76	0.973	0.62	0.971	0.65	0.973	0.61
11	57	1.33	63	1.00	55	1.75	56	1.56	0.961	0.91	0.960	0.96	0.960	0.96
12	68	0.65	63	0.74	74	0.55	74	0.54	0.976	0.52	0.969	0.61	0.980	0.45
13	75	0.61	70	0.67	76	0.59	76	0.58	0.981	0.52	0.975	0.594	0.981	0.51
14	58	0.97	65	0.73	55	1.46	55	1.35	0.968	0.60	0.961	0.72	0.957	0.79
15	54	2.13	52	3.3	55	1.79	56	1.60	0.961	1.01	0.911	1.88	0.963	0.89
16	60	1.16	54	1.6	63	0.98	63	0.96	0.964	0.88	0.962	0.89	0.969	0.79
17	54	1.85	56	1.39	53	2.12	54	1.79	0.960	0.83	0.942	1.13	0.953	0.91
18	57	1.41	65	0.98	56	1.73	56	1.62	0.956	1.10	0.964	0.902	0.957	1.09
19	62	0.87	61	0.89	63	0.82	64	0.77	0.968	0.67	0.965	0.728	0.971	0.63
20	54	1.88	52	2.79	55	1.49	56	1.33	0.950	1.05	0.937	1.16	0.959	0.81
Mean	60.1	1.17	60.8	1.21	60.5	1.21	61	1.15		0.74		0.87		0.72
SD	5.91	0.48	7.15	0.70	7.3	0.54	6.83	0.45		0.204		0.31		0.19

low and vice versa, low estimates of N give high estimates of T.

c) The three methods of the Maximum-Likelihood, the Least-Square t and the Exponential models give almost identical estimates whereas the Least Square x model gives different estimates.

d) The estimates for T, resulting from the Geometric estimators are usually lower than those given by the Standard estimators. The reason for this is that the data is generated randomly according to to the Standard model. When we try to fit a Geometric model to it, we obtain a lower estimate.

e) In spite of the high corrleation between the estimators, it is worthwhile to evaluate all of them, as this gives a wider base for estimating N and T.

4.0 Reliability Models for Delayed Error Correction

The objective of this section is to modify the reliability models and estimators of Section 2 to fit the situation where error correction is delayed. The program is loaded on a tape and each tape version is tested and corrected. While the program is being tested, errors are detected and recorded. Some of these errors, along with some other errors which were detected by other means, are corrected at the end of the test period. The corrections appear on the newer tape version.

Define the following variables:

t_i - time when the i^{th} error is detected. This is the execution time and not the calendar time.

$x_i = t_i - t_{i-1}$ - time between the detection of the $(i-1)^{th}$ error and the i^{th} error.

k - number of tape versions.

n_j - number of errors that were found in the j^{th} tape version.

m_j - number of errors which were corrected in the $(j+1)^{th}$ tape but not in the j^{th} tape.

$M_j = m_1 + m_2 + \ldots m_{j-1}$ - Cumulative number of errors which were corrected in the j^{th} tape version.

N - initial number of errors.

$N_j = N - M_j = N - m_1 - \ldots m_{j-1}$ - number of errors that remain in the j^{th} tape.

I_j - the set of integers which includes the indices of the errors found in the j^{th} tape.

For example, suppose that the program used two tape versions. Four errors were found in the first tape, of which three were

corrected before the second tape was introduced. Three more errors were found in the second tape. The number of tapes here is k=2. The number of errors found in $n_1=4$ and $n_2=3$. The number of errors which were corrected in $m_1=3$. Therefore we have $M_1=0$ and $M_2=3$. The set I_1 includes the numbers [1,2,3,4] and I_2 includes [5,6,7]. Based on the above notations, we modify the models of Section 2 as follows.

4.1 Maximum Likelihood Model

This model can be easily modified to this case as the hazard function is assumed to be proportional to the number of remaining errors. Let the number of errors in the j^{th} tape be N_j, then we have

$$\begin{aligned} N_j &= N - m_1 - m_2 - \cdots - m_{j-1} \\ N_1 &= N \end{aligned} \tag{4-2}$$

Accordingly, the hazard function for the j^{th} tape is

$$z_j = N_j \phi \tag{4-3}$$

The modified model is derived in Appendix B. N, the most likely estimate for N, is the solution of the equation

$$\frac{n \sum_{i=1}^{n} x_i}{\sum_{j=1}^{k} N_j \sum_{i \epsilon I_j} x_i} - \sum_{j=1}^{k} \frac{n_j}{N_j} = 0 \tag{4-4}$$

and $\hat{\phi}$ is derived from

$$\phi = \frac{\sum_{j=1}^{k} \frac{n_j}{N_j}}{\sum_{i=1}^{n} x_i} \tag{4-5}$$

The MTTF for the j^{th} tape, $\hat{T}_j$, is

$$T_j = \frac{1}{\hat{\phi}\hat{N}_j} \tag{4-6}$$

4.2 Geometric Maximum Likelihood Model

When we apply the geometric model we assume that the hazard function decreases geometrically. In this case we can assume two forms of variation of the hazard. The first one is

$$z_j = \lambda_o \, a^j \tag{4-7}$$

According to this model, the hazard decreases due to the correction, independently of the number of errors detected. Another model will be

$$z_j = \lambda_o \, a^{M_j} \tag{4-8}$$

where

$$M_j = m_1 + m_2 + \ldots m_{j-1}$$

and (4-9)

$$M_1 = 0$$

Denote the model of (4-7) as Geometric I, and (4-8) as Geometric II. The derivation of the most likely values of λ_o and a are given in Appendix B. For model I the estimate $\hat{a}$ is the solution of

$$\frac{\sum_{j=1}^{k} (j a^{j} \sum_{i \varepsilon I_j} x_i)}{\sum_{j=1}^{k} j\, n_j} - \sum_{j=1}^{k} a^{j} \sum_{i \varepsilon I_j} x_i = 0 \qquad (4\text{-}10)$$

and $\hat{\lambda}_o$ is found from

$$\lambda_o = \frac{n}{\sum_{j=1}^{k} a^{j} \sum_{i \varepsilon I_j} x_i} \qquad (4\text{-}11)$$

The MTTF for the j^{th} tape is

$$\hat{T}_j = \frac{1}{\lambda_o a^{j}} \qquad (4\text{-}12)$$

For model II the estimate $\hat{a}$ is found from

$$\frac{n \sum_{j=1}^{k} (M_j\, a^{M_j} \sum_{i \varepsilon I_j} x_i)}{\sum_{j=1}^{k} n_j M_j} - \sum_{j=1}^{k} a^{M_j} \sum_{i \varepsilon I_j} x_i = 0 \qquad (4\text{-}13)$$

and $\hat{\lambda}_o$ is determined from

$$\lambda_o = \frac{n}{\sum_{j=1}^{k} (a^{M_j} \sum_{i \varepsilon I_j} x_i)} \qquad (4\text{-}14)$$

Also

$$\hat{T}_j = \frac{1}{\lambda_o\, a^{M_j}} \qquad (4\text{-}15)$$

It can be seen that model I can be obtained as a special case of model II by substituting $M_j = j$

4.3 Least Square x Model

The hazard function z_j, when the j^{th} tape is used, is

$$z_j = \phi N_j \tag{4-16}$$

Accordingly, the mean value of x_i is

$$\bar{x}_i = \frac{1}{\phi N_j} \quad \text{where } i \in I_j. \tag{4-17}$$

The estimation error to be minimized here is

$$E = \sum_{i=1}^{n} (x_i - \bar{x}_i)^2 = \sum_{j=1}^{k} \sum_{i \in I_j} \left(x_i - \frac{1}{\phi N_j}\right)^2 \tag{4-18}$$

The parameters $\hat{N}$ and ϕ which minimize this quantity are derived in Appendix B. $\hat{N}$ is the solution of the equation

$$\frac{\sum_{j=1}^{k} \frac{n_j}{N_j^3}}{\sum_{j=1}^{k} \frac{n_j}{N_j^2}} - \frac{\sum_{j=1}^{k} \left(\frac{1}{N_j^2} \sum_{i \in I_j} x_i\right)}{\sum_{j=1}^{k} \left(\frac{1}{N_j} \sum_{i \in I_j} x_i\right)} = 0 \tag{4-19}$$

and $\hat{\phi}$ is found from

$$\phi = \frac{\sum_{j=1}^{k} \frac{n_j}{N_j^2}}{\sum_{j=1}^{k} \left(\frac{1}{N_j} \sum_{i \in I_j} x_i\right)} \tag{4-20}$$

The MTTF equals

$$\hat{T}_j = \frac{1}{\hat{\phi}\hat{N}_j} \tag{4-21}$$

4.4 Least Square t Model

This model fits the cumulative time, t_i, to its expected value.

Since the hazard function is constant during the use of a certain tape, the mean time between failures will be constatn in that interval too.

$$\bar{x}_i = \frac{1}{N_j \phi} \text{ where } i \epsilon I_j \tag{4-22}$$

To simplify the notation, define

$$N_{(m)} = N_j \text{ where } m \epsilon I_j \tag{4-23}$$

Then we can write

$$\bar{x}_i = \frac{1}{N_{(i)} \phi} \tag{4-24}$$

In view of (4-23), the estimation error, E, equals

$$E = \sum_{i=1}^{n} (t_i - \bar{t}_i)^2 = \sum_{i=1}^{n} (t_i - \sum_{m=1}^{i} \frac{1}{N_{(m)} \phi})^2 \tag{4-25}$$

The estimation of $\hat{N}$ and $\hat{\phi}$ which minimizes (4-25) is given in Appendix B. $\hat{N}$ is the solution of (4-26)

$$\sum_{i=1}^{n} t_i B_i \sum_{i=1}^{n} A_i^2 - \sum_{i=1}^{n} t_i A_i \sum_{i=1}^{n} A_i B_i = 0 \tag{4-26}$$

where

$$A_i = \sum_{m=1}^{k} \frac{1}{N_{(m)}} \tag{4-27}$$

and

$$B_i = \sum_{m=1}^{i} \frac{1}{N_{(m)}^{2}} \tag{4-28}$$

Also, $\hat{\phi}$ is given by

$$\phi = \frac{\sum_{i=1}^{n} A_i^2}{\sum_{i=1}^{n} t_i A_i} \tag{4-29}$$

The MTTF, T_j is

$$\hat{T}_j = \frac{1}{\hat{\phi}\,\hat{N}_j} \tag{4-30}$$

4.5 Geometric Least Square x Model

Section 4.2 presented two forms of the geometric model. Since it was shown that model II is more general, it will be considered in this section and in the following one.

Here again we use the notation

$$M_{(i)} = M_j \text{ where } i \varepsilon I_j \tag{4-31}$$

and recall (4-9)

$$M_j = n_1 + n_2 + \cdot\cdot\cdot\, n_{j-1} \tag{4-32}$$

The objective of the model is to estimate $\hat{a}$ and $\hat{\lambda}_o$ which minimize E.

$$E = \sum_{i=1}^{n} (x_i - \bar{x}_i)^2 = \sum_{i=1}^{n} \left(x_i - \frac{1}{\lambda_o a^{M(i)}}\right)^2 \qquad (4\text{-}33)$$

The estimator $\hat{a}$ is found in Appendix B to be the solution of (4-34)

$$\sum_{i=1}^{n} \frac{x_i M_{(i)}}{a^{M(i)}} \sum_{i=1}^{n} \frac{1}{a^{2M(i)}} - \sum_{i=1}^{n} \frac{x_i}{a^{M(i)}} \sum_{i=1}^{n} \frac{M_{(i)}}{a^{2M(i)}} = 0 \qquad (4\text{-}34)$$

The estimate for $\hat{\lambda}_o$ is given by (4-35)

$$\lambda_o = \frac{\sum_{i=1}^{n} \frac{x_i}{a^{2M(i)}}}{\sum_{i=1}^{n} \frac{x_i}{a^{M(i)}}} \qquad (4\text{-}35)$$

The MTTF is given by

$$\hat{T}_j = \frac{1}{\hat{\lambda}_o a^{M_j}} \qquad (4\text{-}36)$$

4.6 Geometric Least Square t Model

The error resulting from fitting the t_i values with their mean is

$$E = \sum_{i=1}^{n} (t_i - \bar{t}_i)^2 = \sum_{i=1}^{n} \left(t_i - \sum_{m=1}^{i} \frac{1}{\lambda_o a^{M(m)}}\right)^2 \qquad (4.37)$$

The estimate $\hat{a}$ for a is found from

$$\sum_{i=1}^{n} t_i B_i \sum_{i=1}^{n} A_i^2 - \sum_{i=1}^{n} t_i A_i \sum_{i=1}^{n} A_i B_i = 0 \qquad (4\text{-}38)$$

where

$$A_i = \sum_{m=1}^{i} \frac{1}{a^{M_{(m)}}} \tag{4-39}$$

and

$$B_i = \sum_{m=1}^{i} \frac{M_{(m)}}{a^{M_{(m)}}} \tag{4-40}$$

$\hat{\lambda}_o$ is found from:

$$\lambda_o = \frac{\sum_{i=1}^{n} A_i^2}{\sum_{i=1}^{n} t_i A_i} \tag{4-41}$$

5.0 Test and Evaluation of Estimators for the Delayed Correction Case

The modified estimators, given in Section 4 were tested and evaluated. Here again, the objective is to verify that the derivation and the computer program are correct, and to examine the quality of the estimators.

The first part was done by testing the estimators with deterministic data. Instead of generating a sequence of random numbers, x_i, we generated a sequence of the expected values $\bar{x}_i$ for some given parameters N=60, T=1.0. The resulting estimators should equal $\hat{N}$=60 and $\hat{T}$=1.0, if the derivation and the program are correct. Indeed, after some small corrections, all the estimates were equal to the desired values.

The next task of examining the estimators was done in a similar way to the method of Section 3. Random sequences of times were generated to simulate the error detection process with the parameters N=120, n=100 and ϕ=0.05, with a resulting MTTF of T=1.0. 1000 such sequences were generated, and the various estimators were determined for them. The results of the estimators of N are given in the following pages in terms of histograms. These include estimates of N determined by the Maximum-Likelihood estimator and by the Least-Square estimators for both x and t. Note that the "correct" estimate is N=120 and this value is the median for the three histograms. The histograms have the same general shape and are similar to those obtained in Section 3.

SOFTWARE RELIABILITY ANALYSIS PROGRAM. VERSION 2 - PIECEWISE CONSTANT HAZARD F

5 TEST TAPES WITH 100 ERRORS TOTAL - 20 20 20 20 20

HISTOGRAM OF NO EST

JELINSKI MAXIMUM LIKELIHOOD MODEL

DATA PARAMETERS - NO=120 , PHI= .500-01 , SAMPLES=1000

NO EST	FREQ	C.D.F.	PLOT
---	26	26	DID NOT CONVERGE
100	4	30	.XXXX
101	5	35	.XXXXX
102	11	46	.XXXXXXXXXXX
103	19	65	.XXXXXXXXXXXXXXXXXXX
104	25	90	.XXXXXXXXXXXXXXXXXXXXXXXXX
105	19	109	.XXXXXXXXXXXXXXXXXXX
106	16	125	.XXXXXXXXXXXXXXXX
107	20	145	.XXXXXXXXXXXXXXXXXXXX
108	18	163	.XXXXXXXXXXXXXXXXXX
109	27	190	.XXXXXXXXXXXXXXXXXXXXXXXXXXX
110	31	221	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
111	30	251	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
112	44	295	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
113	32	327	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
114	23	350	.XXXXXXXXXXXXXXXXXXXXXXX
115	22	372	.XXXXXXXXXXXXXXXXXXXXXX
116	24	396	.XXXXXXXXXXXXXXXXXXXXXXXX
117	28	424	.XXXXXXXXXXXXXXXXXXXXXXXXXXXX
118	23	447	.XXXXXXXXXXXXXXXXXXXXXXX
119	32	479	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
120	22	501	.XXXXXXXXXXXXXXXXXXXXXX
121	24	525	.XXXXXXXXXXXXXXXXXXXXXXXX
122	29	554	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXX
123	28	582	.XXXXXXXXXXXXXXXXXXXXXXXXXXXX
124	19	601	.XXXXXXXXXXXXXXXXXXX
125	20	621	.XXXXXXXXXXXXXXXXXXXX
126	19	640	.XXXXXXXXXXXXXXXXXXX
127	20	660	.XXXXXXXXXXXXXXXXXXXX
128	17	677	.XXXXXXXXXXXXXXXXX
129	12	689	.XXXXXXXXXXXX
130	8	697	.XXXXXXXX
131	13	710	.XXXXXXXXXXXXX
132	12	722	.XXXXXXXXXXXX
133	21	743	.XXXXXXXXXXXXXXXXXXXXX
134	9	752	.XXXXXXXXX
135	12	764	.XXXXXXXXXXXX
136	14	778	.XXXXXXXXXXXXXX
137	10	788	.XXXXXXXXXX
138	6	794	.XXXXXX
139	13	807	.XXXXXXXXXXXXX
140	12	819	.XXXXXXXXXXXX

141	11	830	.XXXXXXXXXXX
142	4	834	.XXXX
143	7	841	.XXXXXXX
144	4	845	.XXXX
145	7	852	.XXXXXXX
146	4	856	.XXXX
147	9	865	.XXXXXXXXX
148	7	872	.XXXXXXX
149	2	874	.XX
150	2	876	.XX
151	6	882	.XXXXXX
152	3	885	.XXX
153	6	891	.XXXXXX
154	1	892	.X
155	2	894	.XX
156	3	897	.XXX
157	3	900	.XXX
158	6	906	.XXXXXX
159	5	911	.XXXXX
160	4	915	.XXXX
161	7	922	.XXXXXXX
162	3	925	.XXX
163	5	930	.XXXXX
164	3	933	.XXX
165	5	938	.XXXXX
166	4	942	.XXXX
167	2	944	.XX
168	3	947	.XXX
169	1	948	.X
170	1	949	.X
171	4	953	.XXXX
172	1	954	.X
173	1	955	.X
174	5	960	.XXXXX
175	2	962	.XX
177	1	963	.X
178	1	964	.X
184	1	965	.X
185	1	966	.X
186	2	968	.XX
187	1	969	.X
188	4	973	.XXXX
190	3	976	.XXX
191	1	977	.X
192	1	978	.X
193	1	979	.X
195	1	980	.X
197	1	981	.X
199	1	982	.X
201	1	983	.X
203	1	984	.X
205	1	985	.X
206	1	986	.X
209	1	987	.X
217	2	989	.XX
234	1	990	.X
238	1	991	.X

241	1	992	.X
244	1	993	.X
246	1	994	.X
248	1	995	.X
250	1	996	.X
259	1	997	.X
260	1	998	.X
290	1	999	.X
---	1	1000	DID NOT CONVERGE

SOFTWARE RELIABILITY ANALYSIS PROGRAM. VERSION 2 - PIECEWISE CONSTAN

5 TEST TAPES WITH 100 ERRORS TOTAL - 20 20 20 20 20

HISTOGRAM OF NO EST

JELINSKI LEAST SQUARES FIT TO X'S

DATA PARAMETERS - N0=120 , PHI= .500-01 , SAMPLES=1000

NO EST	FREQ	C.D.F.	PLOT
---	37	37	DID NOT CONVERGE
100	5	42	.XXXXX
101	16	58	.XXXXXXXXXXXXXXXX
102	15	73	.XXXXXXXXXXXXXXX
103	14	87	.XXXXXXXXXXXXXX
104	19	106	.XXXXXXXXXXXXXXXXXXX
105	16	122	.XXXXXXXXXXXXXXXX
106	24	146	.XXXXXXXXXXXXXXXXXXXXXXXX
107	30	176	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
108	36	212	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
109	18	230	.XXXXXXXXXXXXXXXXXX
110	19	249	.XXXXXXXXXXXXXXXXXXX
111	27	276	.XXXXXXXXXXXXXXXXXXXXXXXXXXX
112	37	313	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
113	21	334	.XXXXXXXXXXXXXXXXXXXXX
114	23	357	.XXXXXXXXXXXXXXXXXXXXXXX
115	20	377	.XXXXXXXXXXXXXXXXXXXX
116	30	407	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
117	23	430	.XXXXXXXXXXXXXXXXXXXXXXX
118	22	452	.XXXXXXXXXXXXXXXXXXXXXX
119	19	471	.XXXXXXXXXXXXXXXXXXX
120	28	499	.XXXXXXXXXXXXXXXXXXXXXXXXXXXX
121	21	520	.XXXXXXXXXXXXXXXXXXXXX
122	18	538	.XXXXXXXXXXXXXXXXXX
123	28	566	.XXXXXXXXXXXXXXXXXXXXXXXXXXXX
124	18	584	.XXXXXXXXXXXXXXXXXX
125	23	607	.XXXXXXXXXXXXXXXXXXXXXXX
126	17	624	.XXXXXXXXXXXXXXXXX
127	16	640	.XXXXXXXXXXXXXXXX
128	14	654	.XXXXXXXXXXXXXX
129	14	668	.XXXXXXXXXXXXXX
130	17	685	.XXXXXXXXXXXXXXXXX
131	12	697	.XXXXXXXXXXXX
132	8	705	.XXXXXXXX
133	16	721	.XXXXXXXXXXXXXXXX
134	11	732	.XXXXXXXXXXX
135	11	743	.XXXXXXXXXXX
136	8	751	.XXXXXXXX
137	15	766	.XXXXXXXXXXXXXXX
138	12	778	.XXXXXXXXXXXX
139	4	782	.XXXX
140	10	792	.XXXXXXXXXX

SOFTWARE RELIABILITY ANALYSIS PROGRAM. VERSION 2 - PIECEWISE CONST

5 TEST TAPES WITH 100 ERRORS TOTAL - 20 20 20 20 20

HISTOGRAM OF NO EST

JELINSKI LEAST SQUARES FIT TO T'S

DATA PARAMETERS - NO=120 , PHI= .500-01 , SAMPLES=1000

NO EST	FREQ	C.D.F.	PLOT
---	30	30	DID NOT CONVERGE
100	3	33	.XXX
101	13	46	.XXXXXXXXXXXXX
102	15	61	.XXXXXXXXXXXXXXX
103	26	87	.XXXXXXXXXXXXXXXXXXXXXXXXXX
104	18	105	.XXXXXXXXXXXXXXXXXX
105	24	129	.XXXXXXXXXXXXXXXXXXXXXXXX
106	24	153	.XXXXXXXXXXXXXXXXXXXXXXXX
107	25	178	.XXXXXXXXXXXXXXXXXXXXXXXXX
108	23	201	.XXXXXXXXXXXXXXXXXXXXXXX
109	24	225	.XXXXXXXXXXXXXXXXXXXXXXXX
110	26	251	.XXXXXXXXXXXXXXXXXXXXXXXXXX
111	29	280	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXX
112	24	304	.XXXXXXXXXXXXXXXXXXXXXXXX
113	17	321	.XXXXXXXXXXXXXXXXX
114	19	340	.XXXXXXXXXXXXXXXXXXX
115	36	376	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
116	33	409	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
117	16	425	.XXXXXXXXXXXXXXXX
118	26	451	.XXXXXXXXXXXXXXXXXXXXXXXXXX
119	23	474	.XXXXXXXXXXXXXXXXXXXXXXX
120	29	503	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXX
121	15	518	.XXXXXXXXXXXXXXX
122	20	538	.XXXXXXXXXXXXXXXXXXXX
123	18	556	.XXXXXXXXXXXXXXXXXX
124	20	576	.XXXXXXXXXXXXXXXXXXXX
125	21	597	.XXXXXXXXXXXXXXXXXXXXX
126	19	616	.XXXXXXXXXXXXXXXXXXX
127	19	635	.XXXXXXXXXXXXXXXXXXX
128	15	650	.XXXXXXXXXXXXXXX
129	13	663	.XXXXXXXXXXXXX
130	20	683	.XXXXXXXXXXXXXXXXXXXX
131	6	689	.XXXXXX
132	11	700	.XXXXXXXXXXX
133	14	714	.XXXXXXXXXXXXXX
134	16	730	.XXXXXXXXXXXXXXXX
135	12	742	.XXXXXXXXXXXX
136	16	758	.XXXXXXXXXXXXXXXX
137	7	765	.XXXXXXX
138	11	776	.XXXXXXXXXXX
139	7	783	.XXXXXXX
140	12	795	.XXXXXXXXXXXX

Another quantity of interest is the estimate of the MTTF. We have estimated the MTTF from the 1000 sets of data, using all six estimators. The results are presented by some histograms and by Table 5.1. The following page shows a histogram of the MTTF, using the Maximum Likelihood estimator. Note that the histogram is skewed and that the "correct" value of $\hat{T}=1.0$ is the median. This is typical for the Standard estimators, which determine both $\hat{N}$ and $\hat{T}$. This is followed by three histograms of estimates by the Geometric models, and the change is significant. Here we note that the histogram shape resembles the normal distribution curve and that the spread is much smaller than in the previous case. Here again we observe that the mean value of the geometric estimators is considerably below the "correct" value of $\hat{T}=1.0$. The reason for this is that the data was generated according to the Standard model, and when we try to fit a Geometric model to it, we end with a smaller value of $\hat{T}$. The estimator results are analyzed further and the main results are summarized in Table 5.1. These include the number of samples for which the estimate has converged, various percentiles of the estimates and the range of the estimators. The results of Table 5.1 are similar to those of Table 3.1 and they lead to the conclusions made in Section 3, namely, that the estimates of $\hat{T}$ using the Geometric model are generally better than the Standard estimators, and that the LS estimates of $\hat{T}$, based on t are better than those generated from the x data.

SOFTWARE RELIABILITY ANALYSIS PROGRAM. VERSION 2 - PIECEWISE CONSTANT HAZARD FUNCTION

5 TEST TAPES WITH 100 ERRORS TOTAL - 20 20 20 20 20

HISTOGRAM OF TTF EST

JELINSKI MAXIMUM LIKELIHOOD MODEL

DATA PARAMETERS - N0=120 , PHI= .500-01 , SAMPLES=1000

TTF EST	FREQ	C.D.F.	PLOT
.300	2	2	.XX
.350	8	10	.XXXXXXXX
.400	19	29	.XXXXXXXXXXXXXXXXXXX
.450	25	54	.XXXXXXXXXXXXXXXXXXXXXXXXX
.500	40	94	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
.550	54	148	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
.600	40	188	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
.650	41	229	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
.700	50	279	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
.750	40	319	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
.800	53	372	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
.850	44	416	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
.900	34	450	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
.950	39	489	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
1.000	38	527	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
1.050	35	562	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
1.100	28	590	.XXXXXXXXXXXXXXXXXXXXXXXXXXXX
1.150	22	612	.XXXXXXXXXXXXXXXXXXXXXX
1.200	30	642	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
1.250	18	660	.XXXXXXXXXXXXXXXXXX
1.300	19	679	.XXXXXXXXXXXXXXXXXXX
1.350	11	690	.XXXXXXXXXXX
1.400	19	709	.XXXXXXXXXXXXXXXXXXX
1.450	20	729	.XXXXXXXXXXXXXXXXXXXX
1.500	14	743	.XXXXXXXXXXXXXX
1.550	17	760	.XXXXXXXXXXXXXXXXX
1.600	14	774	.XXXXXXXXXXXXXX
1.650	8	782	.XXXXXXXX
1.700	12	794	.XXXXXXXXXXXX
1.750	9	803	.XXXXXXXXX
1.800	9	812	.XXXXXXXXX
1.850	5	817	.XXXXX
1.900	10	827	.XXXXXXXXXX
1.950	7	834	.XXXXXXX
2.000	4	838	.XXXX
2.050	8	846	.XXXXXXXX
2.100	4	850	.XXXX
2.150	3	853	.XXX
2.200	6	859	.XXXXXX
2.250	3	862	.XXX
2.300	4	866	.XXXX
2.400	4	870	.XXXX
2.450	3	873	.XXX
2.500	3	876	.XXX

SOFTWARE RELIABILITY ANALYSIS PROGRAM, VERSION 2 - PIECEWISE CONSTANT HAZARD FUNCTION

5 TEST TAPES WITH 100 ERRORS TOTAL - 20 20 20 20 20

HISTOGRAM OF TTF EST

GEOMETRIC TYPE 2 MAXIMUM LIKELIHOOD MODEL

DATA PARAMETERS - N0=120 , PHI= .500-01 , SAMPLES=1000

TTF EST	FREQ	C.D.F.	PLOT
.150	1	1	.X
.300	4	5	.XXXX
.350	21	26	.XXXXXXXXXXXXXXXXXXXXX
.400	43	69	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
.450	81	150	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
.500	115	265	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
.550	135	400	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
.600	145	545	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
.650	124	669	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
.700	98	767	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
.750	75	842	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
.800	59	901	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
.850	39	940	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
.900	26	966	.XXXXXXXXXXXXXXXXXXXXXXXXXX
.950	12	978	.XXXXXXXXXXXX
1.000	11	989	.XXXXXXXXXXX
1.050	3	992	.XXX
1.100	4	996	.XXXX
1.150	2	998	.XX
1.250	1	999	.X
1.350	1	1000	.X

SOFTWARE RELIABILITY ANALYSIS PROGRAM. VERSION 2 - PIECEWISE CONSTANT HAZARD FUNCTION

5 TEST TAPES WITH 100 ERRORS TOTAL = 20 20 20 20 20

HISTOGRAM OF TTF EST

GEOMETRIC TYPE 2 LEAST SQUARES FIT TO X'S

DATA PARAMETERS = N0=120 , PHI= .500-01 , SAMPLES=1000

TTF EST	FREQ	C.D.F.	PLOT
.150	1	1	.X
.300	3	4	.XXX
.350	17	21	.XXXXXXXXXXXXXXXXX
.400	39	60	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
.450	63	123	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
.500	97	220	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
.550	99	319	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
.600	115	434	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
.650	102	536	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
.700	108	644	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
.750	75	719	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
.800	75	794	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
.850	40	834	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
.900	46	880	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
.950	32	912	.XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
1.000	28	940	.XXXXXXXXXXXXXXXXXXXXXXXXXXXX
1.050	18	958	.XXXXXXXXXXXXXXXXXX
1.100	10	968	.XXXXXXXXXX
1.150	6	974	.XXXXXX
1.200	5	979	.XXXXX
1.250	6	985	.XXXXXX
1.300	4	989	.XXXX
1.350	2	991	.XX
1.400	1	992	.X
1.450	3	995	.XXX
1.500	1	996	.X
1.600	1	997	.X
1.800	1	998	.X
2.100	1	999	.X
2.450	1	1000	.X

```
SOFTWARE RELIABILITY ANALYSIS PROGRAM, VERSION 2 - PIECEWISE CONSTANT HAZARD FUNCTION

 5 TEST TAPES WITH 100 ERRORS TOTAL -   20   20   20   20   20

 HISTOGRAM OF TTF EST

 GEOMETRIC TYPE 2 LEAST SQUARES FIT TO T'S

 DATA PARAMETERS - N0=120 ,  PHI= .500-01 ,  SAMPLES=1000

 TTF EST    FREQ   C.D.F.   PLOT

   .200       1       1    .X
   .250       2       3    .XX
   .300      12      15    .XXXXXXXXXXXX
   .350      25      40    .XXXXXXXXXXXXXXXXXXXXXXXXX
   .400      68     108    .XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   .450      88     196    .XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   .500     107     303    .XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   .550     119     422    .XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   .600     131     553    .XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   .650     123     676    .XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   .700      83     759    .XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   .750      61     820    .XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   .800      74     894    .XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   .850      30     924    .XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   .900      22     946    .XXXXXXXXXXXXXXXXXXXXXX
   .950      18     964    .XXXXXXXXXXXXXXXXXX
  1.000      14     978    .XXXXXXXXXXXXXX
  1.050       5     983    .XXXXX
  1.100       7     990    .XXXXXXX
  1.150       3     993    .XXX
  1.200       3     996    .XXX
  1.250       1     997    .X
  1.300       2     999    .XX
  1.350       1    1000    .X
```

Table 5.1 Summary of results of MTTF estimates for piecewise constant hazard

Method	No. of Samples for Which Estimate Converged (out of 1000)	estimate percentile					estimate range	
		10%	25%	50%	75%	90%	Minimum	Maximum
Maximum Likelihood	973	0.50	0.70	1.00	1.55	3.05	0.30	50.0
Least-square x Model	962	0.50	0.70	1.00	1.60	3.65	0.30	50.0
Geometric Max. Likelihood	1000	0.45	0.45	0.50	0.70	0.80	0.15	1.35
Geometric Least Square x	1000	0.45	0.55	0.65	0.80	0.95	0.15	2.45
Geometric Least Square t	1000	0.40	0.50	0.60	0.70	0.85	0.20	1.55

Another question which interest us is the correlation between estimators. That is, if one estimator produces a large estimate by one method, would the same set of data produce large estimates using the other methods? In order to examine this, we have determined all the six estimators for 20 sets of data. Note that although the data were generated randomly, the same sets of data were applied to all the estimators. The results were the estimates of $\hat{N}$ and $\hat{T}$, by the Standard models, and estimates for $\hat{a}$ and $\hat{T}$ by the Geometric models. All the data points were generated for the parameters N=120, n=100 and ϕ=0.05, with the resulting T=1.0. Thus, the correct values are N=120 and T=1.0. The actual estimates for all the 20 samples are given in Table 5.2

An examination of the results of Table 5.2 leads to the same conclusions derived from Table 3.2. These are:

a) There is a strong correlation between the estimators. When a set of data produces a small estimate of N, it does it with all the estimators (samples 3, 7, 8, 15). Similarly, when the estimates are high, they are high with all the estimators (samples 9, 14, 18, 19).

b) Whenever the estimate of N is high, the estimate of T is low and vice versa, low estimates of N give high estimates of T.

c) The methods of the Maximum-Likelihood and the Least-Square t models give similar estimates whereas the Least Square x model often gives different estimates.

d) The estimates for T, resulting from the Geometric estimators

Table 5.2 Comparison between estimators with identical data samples

Sample Number	Maximum Likelihood		Least Square x Model		Least Square t Model		Geometric Maximum Likelihood		Geometric LS x Model		Geometric LS t Model	
	$\hat{N}$	$\hat{T}$	$\hat{N}$	$\hat{T}$	$\hat{N}$	$\hat{T}$	$\hat{a}$	$\hat{T}$	$\hat{a}$	$\hat{T}$	$\hat{a}$	$\hat{T}$
1	116	0.95	119	0.84	113	1.11	0.985	0.529	0.984	0.567	0.984	0.567
2	120	0.85	120	0.86	115	1.07	0.986	0.514	0.985	0.557	0.986	0.570
3	112	1.52	116	1.26	109	2.01	0.983	0.755	0.984	0.769	0.983	0.798
4	119	1.01	133	0.72	119	1.04	0.983	0.693	0.987	0.603	0.986	0.649
5	115	1.08	112	1.29	118	0.95	0.985	0.581	0.983	0.660	0.986	0.536
6	121	0.99	108	2.00	140	0.64	0.987	0.619	0.983	0.749	0.990	0.491
7	112	1.40	110	1.57	115	1.16	0.985	0.627	0.982	0.757	0.986	0.604
8	112	1.58	112	1.59	115	1.32	0.984	0.782	0.983	0.815	0.986	0.703
9	128	0.73	138	0.62	134	0.66	0.986	0.584	0.988	0.527	0.988	0.525
10	116	1.08	116	1.13	113	1.29	0.986	0.556	0.983	0.679	01985	0.608
11	120	0.97	130	0.75	118	1.05	0.985	0.632	0.987	0.596	0.985	0.650
12	123	0.72	125	0.68	130	0.61	0.986	0.477	0.986	0.481	0.988	0.445
13	122	0.88	122	0.88	108	1.90	0.987	0.567	0.986	0.614	0.983	0.708
14	132	0.66	124	0.77	130	0.67	0.989	0.453	0.987	0.536	0.989	0.460
15	114	1.25	115	1.19	111	1.60	0.985	0.633	0.983	0.723	0.984	0.695
16	118	1.02	119	0.98	119	0.99	0.986	0.577	0.984	0.665	0.986	0.575
17	112	1.00	109	1.94	117	1.23	0.984	0.778	0.982	0.861	0.986	0.574
18	140	0.75	138	0.77	142	0.72	0.990	0.591	0.989	0.613	0.990	0.564
19	128	0.81	128	0.81	127	0.85	0.988	0.561	0.987	0.611	0.987	0.593
20	112	1.74	123	1.14	116	1.47	0.983	0.938	0.984	0.862	0.985	0.328
Mean	119.6	1.08	120.3	1.09	120.5	1.12	0.9857	0.622	0.9849	0.662	0.9844	0.607
SD	7.6	0.30	9.1	0.41	9.9	0.40	0.0019	0.117	0.0021	0.112	0.0083	0.102

are usually lower than those given by the Standard estimators. The reason for this is that the data is generated randomly according to the Standard model. When we try to fit a Geometric model to it, we obtain a lower estimate.

e) In spite of the high correlation between the estimators, it is worthwhile to evaluate all of them, as this gives a wider base for estimating N and T.

6.0 Accuracy of Estimates

In addition to estimating the parameters $\hat{N}$ and $\hat{T}$, we wish to determine the accuracy of the results. This is done by two different methods. The first method is the development of confidence intervals for the estimators, and the second method is the examination of the effect of N, the initial number of errors, and n, the number of detected errors, on the accuracy of the estimators.

In order to simply the analysis, we discuss only the Instant Correction case, where errors are corrected immediately after detection.

Two methods can be used for the development of confidence intervals. The first one is based on the fact that maximum likelihood estimators which are based on large samples of data are normally distributed, with the true value as a mean. The resulting confidence interval is called "large sample confidence interval." The second method for constructing confidence intervals is a general one and does not rely on the assumption of normal distribution. The two methods are described in [10]. For the purpose of completeness, we present the two methods in Sections 6.1 and 6.2.

The second method of evaluating the effect of N and n on the accuracy of the estimator is given in Section 6.3.

6.1 Large Sample Confidence Intervals

This method is based on the assumption that the sample size is large enough to result in normally distributed estimators $\hat{N}$ and $\hat{T}$. This assumption is accepted by researchers [12] for samples

of size $n > 30$.

The method involves two steps: the evaluation of the variance, and the construction of the confidence interval. The variance can be calculated by the method which is given in Appendix C. It is found there that the variance of $\hat{N}$ is:

$$\mathrm{Var}\,(\hat{N}) = \frac{n}{Sn - A^2\,\hat{\phi}^2} \tag{6-1}$$

where

$$A = \sum_{i=1}^{n} x_i \tag{6-2}$$

and

$$S = \sum_{i=1}^{n} \frac{1}{(\hat{N}-i+1)^2} \tag{6-3}$$

Similarly, the variance of $\hat{T}$ is found to be:

$$\mathrm{Var}\,(\hat{T}) = \frac{1}{\Delta}\,\left(S - \frac{n}{(\hat{N}-n)^2} + \frac{2B}{(\hat{N}-n)^3\,\hat{T}}\right) \tag{6-4}$$

where S is given by (6-3) and

$$B = \sum_{i=1}^{n} (n-i+1)\,x_i \tag{6-5}$$

Also

$$\Delta = \left(S - \frac{n}{(\hat{N}-n)^2} + \frac{2B}{(\hat{N}-n)^3\,\hat{T}}\right)\left(\frac{-n}{\hat{T}^2} + \frac{2A}{\hat{T}^3} + \frac{2B}{(\hat{N}-n)\,\hat{T}^3}\right) - \frac{B^2}{(\hat{N}-n)^4\,\hat{T}^4} \tag{6-6}$$

Once the variance is determined the confidence interval is given by

$$N \pm \lambda_{\left(\frac{1-\gamma}{2}\right)} \sqrt{\mathrm{Var}(\hat{N})} \tag{6-7}$$

and

$$\hat{T} \pm \lambda_{(\frac{1-\gamma}{2})} \sqrt{Var(\hat{T})} \tag{6-8}$$

where γ is the confidence level and λ_α is the number of standard deviations which are exceeded with probability α. In case that we want a one-sided confidence interval, we modify (6-7) and 6-8). The one-sided limits will be

$$\hat{N} + \lambda_{(1-\gamma)} \sqrt{Var(\hat{N})} \tag{6-9}$$

and

$$\hat{T} - \lambda_{(1-\gamma)} \sqrt{Var(\hat{T})} \tag{6-10}$$

6.2 General Confidence Intervals

The method used in the preceding section is based on the assumption that a large number of errors were corrected. Here we present a general method which does not require large samples of errors. The method, which is given by [10], is described in Appendix D. We present here a brief description of the method.

Suppose that we estimate the number of errors to be $\hat{N}'$, and that this is based on data from n' points. Our objective is to construct a confidence interval. For definiteness let the desired confidence level be 90 percent. The method is based on finding two numbers, N_1 and N_2. N_1 is such a number that when the true number of errors is N_1, 5 percent of the estimates are below $\hat{N}'$. Similarly, N_2 is such that when N is equal to N_2, 5 percent of the estimates are above $\hat{N}'$. These values form a confidence interval (N_2, N_1) as discussed in Appendix D.

The detailed procedure for using this method is illustrated next for the cases $n' = 60$ and $n' = 100$. The first step is to construct the percentile curves, as shown in Fig. 6.1 and 6.2 The point A, on Fig. 6.1, indicates that if the initial number of errors is N=90, and $n' = 60$ of those errors were corrected, then 10% of the estimates, $\hat{N}$, will be below 73 and 90% will be above it. Similarly, point B shows that 75% of the estimates in this case are above 79. One can conclude that 15% of the estimates will be within 73 and 79.

In order to determine the points A and B we start with the initial number of errors N=90. We can choose ϕ to be any positive constant, as it was shown in Appendix D that ϕ does not influence the confidence interval. Next simulate the error detection process and generate $n' = 60$ interdetection times x_i, according to the probability density function (1-2). Based on this sequence, we estimate $\hat{N}$ and record its value. This process of generating a sequence and estimating $\hat{N}$ is repeated 1000 times and the resulting estimates $\hat{N}$ are represented by a histogram. It was found from the histogram that 10% of the estimates $\hat{N}$ are below 73, this is the basis for the construction of the point A.

Next, we change the initial number of errors to N=80, and repeat the process. When we obtain enough percentile points, we can join them to form the curves of Fig. 6.1 and Fig. 6.2.

The percentile curves, along with the estimate $\hat{N}$, allow us to construct a confidence interval of any desired level. Furthermore, the interval may be one-sided or two-sided. For example, if the estimate is $\hat{N}=120$ for the case $n'=100$, we can see from Fig. 6.2 that the 5% curve intersects the $\hat{N}=120$ line at N=145, forming a 95%

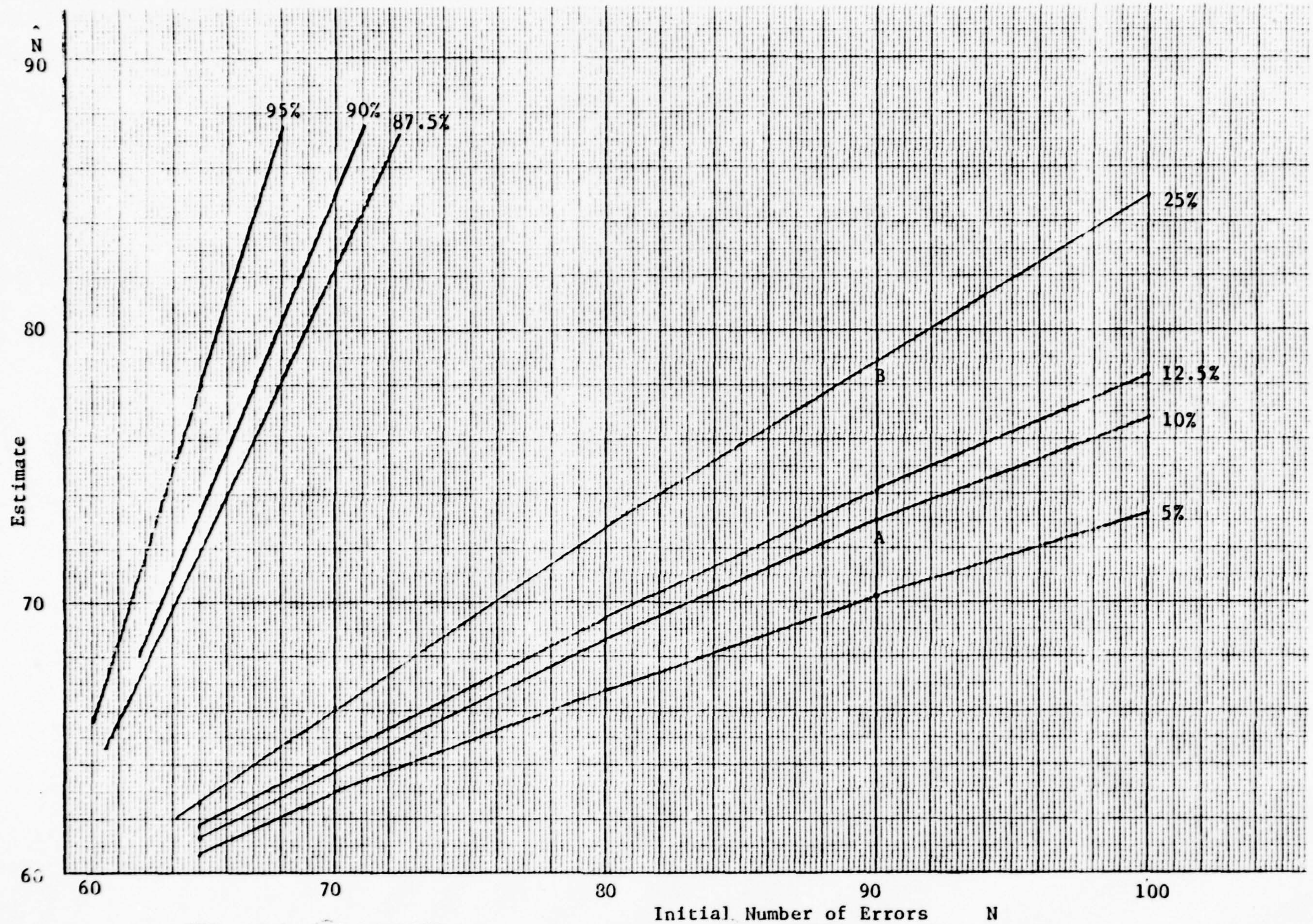

Figure 6.1. Percentile curves for confidence intervals. n' = 60.

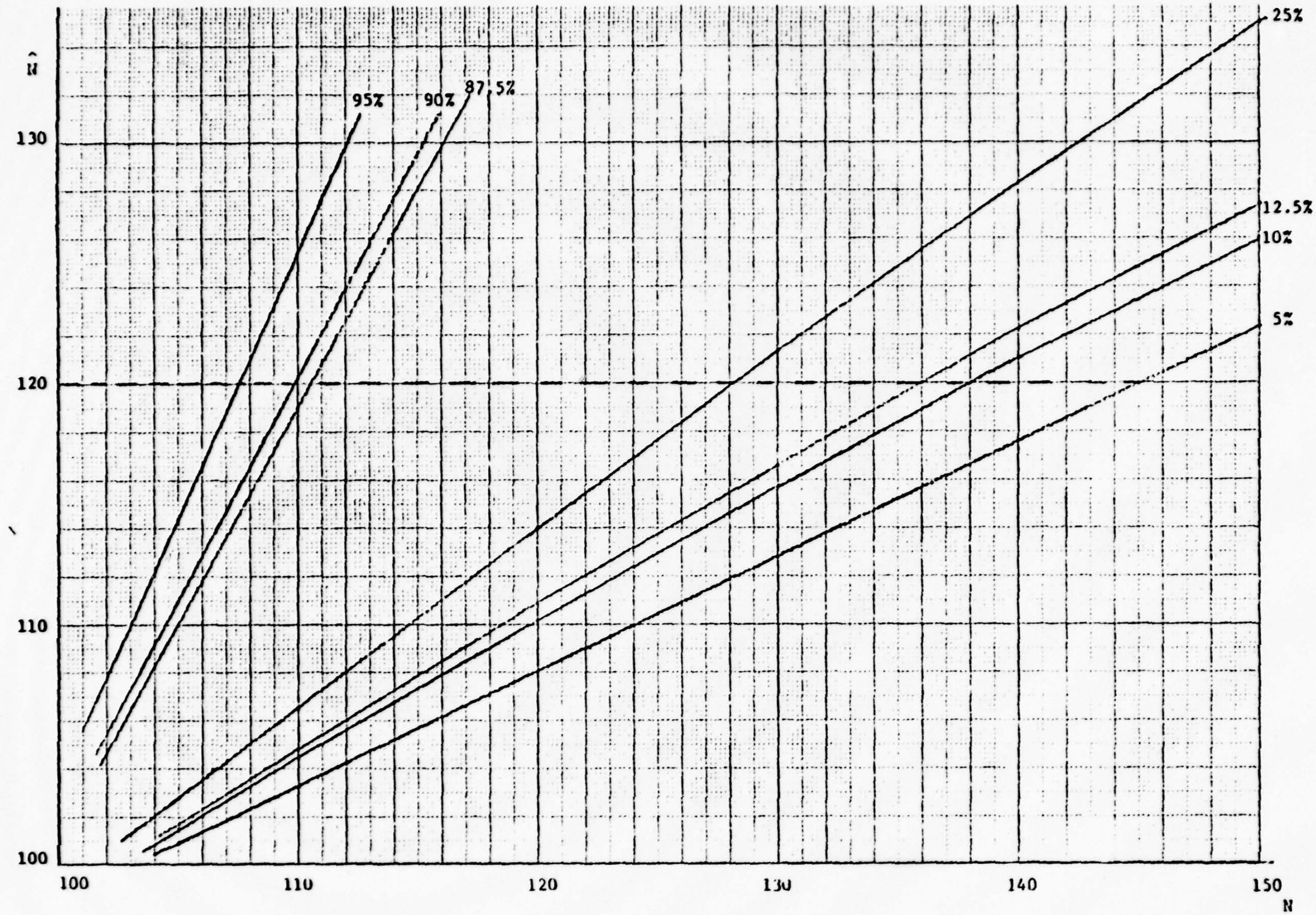

Figure 6.2. Percentile curves for confidence intervals. n' = 100.

one-tailed interval. In other words, there is a 95% probability that the true number of initial errors is below 145. Similarly, the probability is 90% that N is below 138. One can construct a two-tailed interval by considering the upper and the lower limits. Thus, the probability is 90% that N is between 145 and 107. Also, the probability is 75% that N is between 136 and 110. The two tailed intervals do not have to be symmetric. Consequently, the probability is 85% that N is between $110 < N < 145$ or $107 < N < 136$.

Note that in the actual testing $\hat{N}$ is known and therefore we need the percentile curves only around that level of $\hat{N}$. This reduces the amount of work considerably.

This method is suitable for estimates with one unknown parameter, such as $\hat{N}$ which depends only on N. If the estimator is a function of two parameters, as in the case for $\hat{T}$, being dependent on T and N, this method becomes quite complex and it is not recommended.

6.3 Effect of N and n on Estimator Accuracy

An alternative approach for describing the accuracy of the estimator is by studying the effect of N, the initial number of errors, and n, the number of the detected errors, on the accuracy of the estimator. Here again we rely on the results of Appendix D which shows that the estimator $\hat{N}$ is independent of the parameter ϕ.

The first step in the evaluation of the accuracy is to define an error term, E, which describes the inaccuracy of the estimate. Here we select E as

$$E = [\text{Expected value of } (\hat{N}-N)^2]^{1/2} \tag{6-11}$$

The expected value is approximated by the average value over 1000 samples. In case that some sample resulted in a divergent estimate, we assign the maximum value of $\hat{N} = 1000$ to that sample. Next, the error E is determined for various values of N and n. The results are given by Table 6.1.

Table 6.1. Estimation error as a function of N and N-n.

N-n \ N	50	75	100	150	200
2	3.9	--	--	--	--
4	8.9	--	--	--	--
5	19.9	5.95	4.7	3.9	3.85
10	79.3	11.0	8.5	6.2	5.6
15	202.0	62.5	11.3	9.0	7.6
20	328.0	96.7	24.2	11.4	9.9
25	--	147.0	59.3	15.3	--
30	--	219.0	93.4	22.5	14.3
40	--	--	194.0	40.1	20.1
50	--	--	299.0	73.2	29.6
60	--	--	--	140.0	41.4
80	--	--	--	296.0	98.4
100	--	--	--	--	213.0

The same results are illustrated graphically by Fig. 6.3. An alternative way to represent the results of Table 6.1 is by finding

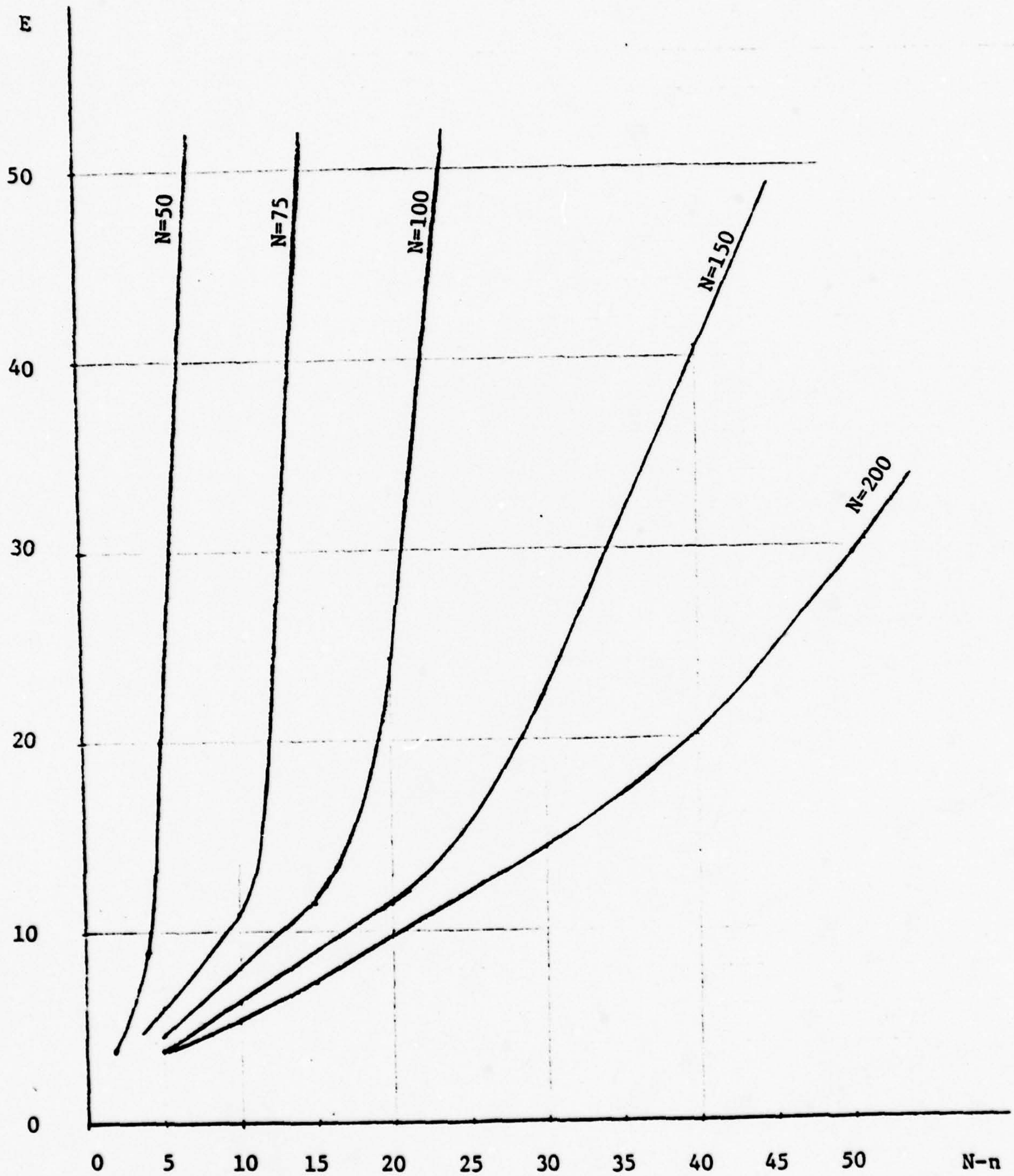

Fig. 6.3 The estimation error as a function of N, the initial number of errors, and the number of the remaining errors, N-n.

curves of constant estimate errors as functions of N and N-n. This is shown in Fig. 6.4.

Figure 6.3 indicates clearly that there is a "knee" in the error curve for values of N, beyond which the estimate error becomes very large. This knee occurs at N-n=0 for the curve N=75 and moves to N-n=25 for N=150. This information may be useful in evaluating the quality of the estimate. For example, let the number of the detected errors, n=60 and the estimate $\hat{N}$ equals 100. If we assume that the accuracy of the estimate is good and therefore $N \cong \hat{N}$, the resulting N-n will be approximately 40, and from Fig. 6.3 we see that for this condition the error is very large, and the estimate is of no practical value. On the other hand, if the number of the detected errors is n=145 and the estimate is $\hat{N}$=150, the accuracy of the result is probably good.

Figure 6.4 reveals another interesting point. It shows that as N increases, the accuracy of the estimator improves significantly, even if the number of remaining errors, N-n, remains the same.

The results of this section confirm the intuitive feeling that the estimator accuracy improves as N increases or as N-n decreases. But it goes beyond that by providing a quantitative measure for E, as shown in Fig. 6.3.

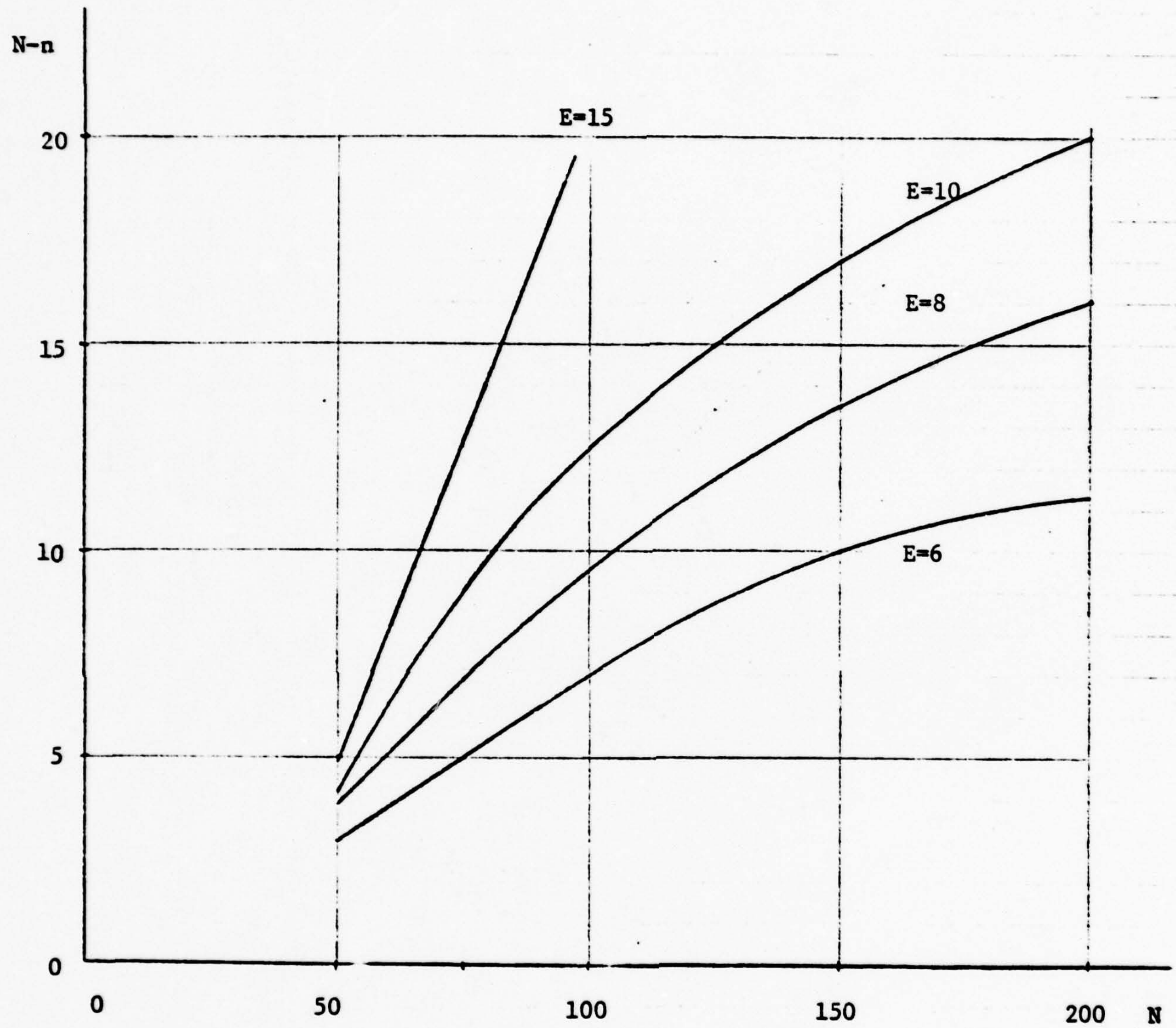

Fig. 6.4. Curves of equal estimate errors, as functions of N and n.

7.0 Conclusions

Several methods were developed for estimating N, the number of errors in a program, and T, the MTTF. The estimators form two groups: The Standard estimators and the Geometric ones. The Standard estimators determine both N and T whereas the Geometric estimators can evaluate only T. The various estimators were tested and evaluated. It was found that the various estimators are strongly correlated but they differ enough to justify generating all of them.

The estimators were later modified to fit the case where errors correction is delayed, as may be required in our case. Test results indicate essentially the same behavior as in the case of Instant Correction.

In addition to estimating N and T, we can learn about the accuracy of the estimates. This can be done by constructing confidence intervals by one of the two methods suggested in this study or by observing the effects of N and n on the estimate accuracy, as discussed in Section 6.

Appendix A-Derivation of Model Equations

A.1 Maximum Likelihood

The probability density of x_i is

$$f(x_i) = \phi(N-i+1)\ e^{-\phi(N-i+1)\ x_i} \qquad i = 1, 2 \ldots n \qquad \text{(A1)}$$

The likelihood, L is

$$L(x_1 x_2 \ldots x_n) = \prod_{i=1}^{n} f(x_i) = \prod_{i=1}^{n} (N-i+1)\ e^{-\phi(N-i+1)x_i} \qquad \text{(A2)}$$

In order to maximize the likelihood, we may maximize ln(L).

$$\ln L(x_1 \ldots x_n) = \sum_{i=1}^{n} [\ln\phi + \ln(N-i+1) - \phi(N-i+1)\ x_i]$$

$$= n \ln\phi + \sum_{i=1}^{n} \ln(N-i+1) - \phi \sum_{i=1}^{n} (N-i+1)\ x_i \qquad \text{(A3)}$$

Now require:

$$\frac{\partial \ln L}{\partial N} = \sum_{i=1}^{n} \frac{1}{N-i+1} - \phi \sum_{i=1}^{n} x_i = 0 \qquad \text{(A4)}$$

$$\frac{\phi \ln L}{\partial \phi} = \frac{n}{\phi} - \sum_{i=1}^{n} (N-i+1)\ x_i = 0 \qquad \text{(A5)}$$

Rewrite (A4) as

$$\phi = \frac{\sum_{i=1}^{n} \frac{1}{N-i+1}}{\sum_{i=1}^{n} x_i} \qquad \text{(A6)}$$

Now substitute (A6) in (A5) and rearrange

$$\sum_{i=1}^{n} \frac{1}{N-i+1} = \frac{n}{N - \frac{\sum_{i=1}^{n} (i-1)\, x_i}{\sum_{i=1}^{n} x_i}} \qquad \text{(A7)}$$

(A7) and (A6) will be used for estimating N and ϕ respectively

A.2 Geometric Maximum Likelihood Model

The probability density function of x_i is given by

$$f(x_i) = \lambda_o\, a^i\, e^{-\lambda_o a^i x_i} \qquad \text{(A8)}$$

The likelihood function is

$$L(x_1 \ldots x_n) = \prod_{i=1}^{n} \lambda_o a^i \; e^{-\lambda_o a^i x_i} \qquad \text{(A9)}$$

$$\ln L(x_1 \ldots x_n) = \sum_{i=1}^{n} [\ln \lambda_o + i \ln a - \lambda_o a^i x_i] =$$

$$= n \ln \lambda_o + \frac{n\,(n+1)}{2} \ln a - \lambda_o \sum_{i=1}^{n} a^i x_i \qquad \text{(A10)}$$

In order to maximize the likelihood, require

$$\frac{\partial \ln L}{\partial a} = \frac{n(n+1)}{2a} - \lambda_o \sum_{i=1}^{n} i\, a^{i-1} x_i = 0 \qquad \text{(A11)}$$

$$\frac{\partial \ln L}{\partial \lambda_o} = \frac{n}{\lambda_o} - \sum_{i=1}^{n} a^i x_i = 0 \qquad \text{(A12)}$$

Rewrite (A12) as

$$\lambda_o = \frac{n}{\sum_{i=1}^{n} a^i x_i} \qquad \text{(A13)}$$

And substitute (A13) in (A11) to form:

$$\frac{n(n+1)}{2a} - \frac{n \sum_{i=1}^{n} i \, a^{i-1} x_i}{\sum_{i=1}^{n} a^{i} x_i} = 0 \tag{A14}$$

Solve (A14) for $\hat{a}$ and (A13) for $\hat{\lambda}_o$.

A.3 Least Square - x Model

The error to be minimized is

$$E = \sum_{i=1}^{n} \left(x_i - \frac{1}{\phi(N-i+1)}\right)^2 \tag{A15}$$

Require

$$\frac{\partial E}{\partial \phi} = 2 \sum_{i=1}^{n} \left[\left(x_i - \frac{1}{\phi(N-i+1)}\right) \cdot \frac{1}{\phi^2 \, (N-i+1)}\right] = 0 \tag{A16}$$

$$\frac{\partial E}{\partial N} = 2 \sum_{i=1}^{n} \left[\left(x_i - \frac{1}{\phi(N-i+1)}\right) \frac{1}{\phi(N-i+1)^2}\right] = 0 \tag{A17}$$

Rewrite (A16 and (A17) as

$$\phi \sum_{i=1}^{n} \frac{x_i}{(N-i+1)} - \sum_{i=1}^{n} \frac{1}{(N-i+1)^2} = 0 \tag{A18}$$

$$\phi \sum_{i=1}^{n} \frac{x_i}{(N-i+1)^2} - \sum_{i=1}^{n} \frac{1}{(N-i+1)^3} = 0 \tag{A19}$$

Express ϕ as

$$\phi = \frac{\sum_{i=1}^{n} \frac{1}{(N-i+1)^2}}{\sum_{i=1}^{n} \frac{x_i}{N-i+1}} \tag{A20}$$

and substitute (A20) in (A19) to form

$$\sum_{i=1}^{n} \frac{x_i}{(N-i+1)^2} \sum_{i=1}^{n} \frac{1}{(N-i+1)^2} - \sum_{i=1}^{n} \frac{x_i}{(N-i+1)} \sum_{i=1}^{n} \frac{1}{(N-i+1)^3} = 0 \tag{A21}$$

Next, solve (A21) for $\hat{N}$ and determine $\hat{\phi}$ from (A20)

A.4 Least Square t Model

The error here is

$$E = \sum_{i=1}^{n} (t_i - \bar{t}_i)^2 = \sum_{i=1}^{n} (t_i - \sum_{j=1}^{i} \frac{1}{\phi(N-j+1)})^2 \tag{A22}$$

$$\text{where } t_i = \sum_{j=1}^{n} x_j \tag{A23}$$

In order to minimize E, require:

$$\frac{\partial E}{\partial N} = 2 \sum_{i=1}^{n} [(t_i - \sum_{j=1}^{i} \frac{1}{\phi(N-j+1)}) \sum_{j=1}^{i} \frac{1}{\phi(N-j+1)^2}] = 0 \tag{A24}$$

$$\frac{\partial E}{\partial \phi} = 2 \sum_{i=1}^{n} [(t_i - \sum_{j=1}^{i} \frac{1}{\phi(N-j+1)}) \sum_{j=1}^{i} \frac{1}{\phi^2(N-j+1)}] = 0 \tag{A25}$$

Rewrite (A24) and (A25) as

$$\phi \sum_{i=1}^{n} [t_i \sum_{j=1}^{i} \frac{1}{(N-j+1)^2}] - \sum_{i=1}^{n} [\sum_{j=1}^{i} \frac{1}{(N-j+1)} \cdot \sum_{j=1}^{i} \frac{1}{(N-j+1)^2}] = 0 \tag{A26}$$

$$\phi \sum_{i=1}^{n}[t_i \sum_{j=1}^{i} \frac{1}{N-j+1}] - \sum_{i=1}^{n} [\sum_{j=1}^{i} \frac{1}{N-j+1}]^2 = 0 \tag{A27}$$

Now express ϕ as

$$\phi = \frac{\sum_{i=1}^{n} (\sum_{j=1}^{i} \frac{1}{(N-j+1)})^2}{\sum_{i=1}^{n} (t_i \sum_{j=1}^{i} \frac{1}{N-j+1})} \tag{A28}$$

And substitute (A28) in (A26) to form:

$$\sum_{i=1}^{n} (t_i \sum_{j=1}^{i} \frac{1}{(N-j+1)^2}) \cdot \sum_{i=1}^{n} (\sum_{j=1}^{i} \frac{1}{(N-j+1)})^2 -$$

$$- \sum_{i=1}^{n} (t_i \sum_{j=1}^{i} \frac{1}{N-j+1}) \cdot \sum_{i=1}^{n} (\sum_{j=1}^{i} \frac{1}{(N-j+1)} \cdot \sum_{j=1}^{i} \frac{1}{(N-j+1)^2}) = 0 \tag{A29}$$

In order to simplify the expressions define

$$A_i = \sum_{j=1}^{i} \frac{1}{N-j+1} \tag{A30}$$

$$B_i = \sum_{j=1}^{i} \frac{1}{(N-j+1)^2} \tag{A31}$$

Then we may write (A29) as

$$\sum_{i=1}^{n} (t_i B_i) \sum_{i=1}^{n} A_i^2 - \sum_{i=1}^{n} t_i A_i \sum_{i=1}^{n} A_i B_i = 0 \tag{A32}$$

Also, (A28) will become:

$$\phi = \frac{\sum_{i=1}^{n} A_i^2}{\sum_{i=1}^{n} t_i A_i} \tag{A33}$$

Eq. (A32) is solved first for $\hat{N}$, and (A33) is used to derive $\hat{\phi}$.

A.5 Geometric Least Square x Model

The estimation error is

$$E = \sum_{i=1}^{n} (x_i - \frac{1}{\lambda_o a^i})^2 \tag{A34}$$

and the objective is to determine $\hat{\lambda}_o$ and $\hat{a}$ which minimize E.

Require

$$\frac{\partial E}{\partial a} = 2 \sum_{i=1}^{n} [(x_i - \frac{1}{\lambda_o a^i}) \cdot \frac{i}{\lambda_o a^{i+1}}] = 0 \tag{A35}$$

$$\frac{\partial E}{\partial \lambda_o} = 2 \sum_{i=1}^{n} [(x_i - \frac{1}{\lambda_o a^i}) \cdot \frac{1}{\lambda_o^2 a^i}] = 0. \tag{A36}$$

Rewrite (A35) and (A36) as

$$\lambda_o \sum_{i=1}^{n} \frac{i\, x_i}{a^i} - \sum_{i=1}^{n} \frac{i}{a^{2i}} = 0 \tag{A37}$$

$$\lambda_o \sum_{i=1}^{n} \frac{x_i}{a^i} - \sum_{i=1}^{n} \frac{1}{a^{2i}} = 0 \tag{A38}$$

Now, express λ_o as

$$\lambda_o = \frac{\sum_{i=1}^{n} \frac{1}{a^{2i}}}{\sum_{i=1}^{n} \frac{x_i}{a^i}} \tag{A39}$$

Sub. (A39) in (A37) and re-arrange to obtain

$$\sum_{i=1}^{n} \frac{i\, x_i}{a^i} \cdot \sum_{i=1}^{n} \frac{1}{a^{2i}} - \sum_{i=1}^{n} \frac{x_i}{a^i} \cdot \sum_{i=1}^{n} \frac{i}{a^{2i}} = 0 \tag{A40}$$

A.6 Geometric Least Square t Model

The estimation error in this case is

$$E = \sum_{i=1}^{n} \left(t_i - \sum_{j=1}^{i} \frac{1}{\lambda_o\, a^i}\right)^2 \tag{A41}$$

The objective is to find $\hat{\lambda}_o$ and $\hat{a}$ which minimize E. Require:

$$\frac{\partial E}{\partial a} = 2 \sum_{i=1}^{n} \left[\left(t_i - \sum_{j=1}^{i} \frac{1}{\lambda_o\, a^j}\right) \left(\sum_{j=1}^{i} \frac{j}{\lambda_o\, a^{j+1}}\right)\right] = 0 \tag{A42}$$

and

$$\frac{\partial E}{\partial \lambda_o} = 2 \sum_{i=1}^{n} \left[\left(t_i - \sum_{j=1}^{i} \frac{1}{\lambda_o\, a^j}\right)\left(\sum_{j=1}^{i} \frac{1}{\lambda_o^2\, a^j}\right)\right] = 0 \tag{A43}$$

Rewrite (A42) and (A43) as

$$\lambda_o \sum_{i=1}^{n} t_i \left(\sum_{j=1}^{i} \frac{j}{a^{j+1}}\right) - \sum_{i=1}^{n} \left(\sum_{j=1}^{i} \frac{1}{a^j} \sum_{j=1}^{i} \frac{j}{a^{j+1}}\right) = 0 \tag{A44}$$

$$\lambda_o \sum_{i=1}^{n} t_i \left(\sum_{j=1}^{i} \frac{1}{a^j}\right) - \sum_{i=1}^{n} \left(\sum_{j=1}^{i} \frac{1}{a^j} \sum_{j=1}^{i} \frac{1}{a^j}\right) = 0 \tag{A45}$$

Define the following functions

$$C_i = \sum_{j=1}^{i} \frac{1}{a^j} \tag{A46}$$

$$D_i = \sum_{j=1}^{i} \frac{j}{a^j} \tag{A47}$$

and re-write (A44) and (A45)

$$\lambda_o \sum_{i=1}^{n} t_i D_i - \sum_{i=1}^{n} C_i D_i = 0 \tag{A48}$$

$$\lambda_o \sum_{i=1}^{n} t_i C_i - \sum_{i=1}^{n} C_i^2 = 0 \tag{A49}$$

Now, derive λ_o from (A49)

$$\lambda_o = \frac{\sum_{i=1}^{n} C_i^2}{\sum_{i=1}^{n} t_i C_i} \tag{A50}$$

and substitute (A50) in (A48)

$$\sum_{i=1}^{n} t_i D_i \cdot \sum_{i=1}^{n} C_i^2 - \sum_{i=1}^{n} t_i C_i \sum_{i=1}^{n} C_i D_i = 0 \tag{A51}$$

A.7 Exponential Least Square Model

The estimation error here is

$$E = \sum_{i=1}^{n} [t_i + \frac{1}{\phi} \ln (\frac{N-i}{N})]^2 \tag{A52}$$

In order to minimize E, require:

$$\frac{\partial E}{\partial \phi} = 2 \sum_{i=1}^{n} [(t_i + \frac{1}{\phi} \ln \frac{N-i}{N})(\frac{-1}{\phi 2} \cdot \ln (\frac{N-i}{N}))] = 0. \tag{A53}$$

$$\frac{\partial E}{\partial N} = 2 \sum_{i=1}^{n} [(t_i + \frac{1}{\phi} \ln \frac{N-i}{N}) \frac{1}{\phi} \frac{i}{N(N-i)}] = 0. \tag{A54}$$

Rearrange (A53) as

$$\phi \sum_{i=1}^{n} t_i \ln (\frac{N-i}{N}) + \sum_{i=1}^{n} \ln^2 (\frac{N-i}{N}) = 0 \tag{A55}$$

Then

$$\phi = \frac{-\sum_{i=1}^{n} \ln^2 (\frac{N-i}{N})}{\sum_{i=1}^{n} t_i \ln (\frac{N-i}{N})} \tag{A56}$$

Sub. (A56) in (A54)

$$\sum_{i=1}^{n} \frac{i\, t_i}{N-i} \sum_{i=1}^{n} \ln^2 (\frac{N-i}{N}) - \sum_{i=1}^{n} t_i \ln (\frac{N-i}{N}) \cdot \sum_{i=1}^{n} \frac{i}{N-i} \ln (\frac{N-i}{N}) = 0 \tag{A57}$$

Eq. (A57) is solved for $\hat{N}$ and $\hat{\phi}$ is determined from (A56).

Appendix B--Derivation of Model Equations for Piecewise Constant Hazard

The equations for the modified models of Section 4 are derived in this appendix.

B.1 Maximum Likelihood Model

The probability density function for x_i is

$$f(x_i) = \phi N_j \quad e^{-\phi N_j x_i} \tag{B1}$$

where j is the index of the tape on which the i^{th} error was discovered.

The likelihood function is

$$L(x_1 x_2 \; . \; . \; . \; x_n) = \prod_{i \varepsilon I_1} N_1 \phi \; e^{-N_1 \phi x_i} \ldots . \prod_{i \varepsilon I_k} N_k \phi \; e^{-N_k \phi x_i} \tag{B2}$$

$$\ln L \; (x_1 x_2 \; . \; . \; . \; x_n) = \sum_{j=1}^{k} \; \sum_{i \varepsilon I_j} \; (\ln\phi \; + \ln N_j - \phi N_j \; x_i) \tag{B3}$$

In order to maximize the likelihood require:

$$\frac{\partial \; \ln L}{\partial \; N} = \sum_{j=1}^{k} \; \sum_{i \varepsilon I_j} \; \left(\frac{1}{N_j} - \phi x_i\right) = 0 \tag{B4}$$

or

$$\sum_{j=1}^{k} \frac{n_j}{N_j} - \phi \sum_{i=1}^{n} x_i = 0 \tag{B5}$$

Similarly require

$$\frac{\partial \ln L}{\partial \phi} = \sum_{j=1}^{k} \; \sum_{i \varepsilon I_j} \; \left(\frac{1}{\phi} - N_j \; x_i\right) = 0 \tag{B6}$$

or

$$n - \phi \sum_{j=1}^{k} N_j \sum_{i\epsilon I_j} x_i = 0 \tag{B7}$$

From (B5) we obtain

$$\phi = \frac{\sum_{j=1}^{k} \frac{n_j}{N_j}}{\sum_{i=1}^{n} x_i} \tag{B8}$$

Then substitute (B8) in (B7) to form:

$$\frac{n \sum_{i=1}^{n} x_i}{\sum_{j=1}^{k} N_j \sum_{i\epsilon I_j} x_i} - \sum_{j=1}^{k} \frac{n_j}{N_j} = 0 \tag{B9}$$

Then $\hat{N}$ is the solution of (B9) and $\hat{\phi}$ is derived from (B8)

B.2 Geometric Maximum--Likelihood Model

Consider first the derivation of model I.

$$z_j = \lambda_o a^j \tag{B10}$$

The likelihood function is

$$L(x_1 x_2 \ldots x_n) = \prod_{i\epsilon I_1} \lambda_o a e^{-\lambda_o a x_i} \prod_{i\epsilon I_2} \lambda_o a^2 e^{-\lambda_o a^2 x_i} \ldots \prod_{i\epsilon I_k} \lambda_o a^k e^{-\lambda_o a^k x_i}$$

and

$$\ln L(x_1 \ldots x_n) = \sum_{j=1}^{k} \sum_{i\epsilon I_j} \ln \lambda_o + j \ln a - \lambda_o a^j x_i =$$

$$= n \ln \lambda_o + \ln a \sum_{j=1}^{k} j n_j - \lambda_o \sum_{j=1}^{k} a^j \sum_{i\epsilon I_j} x_i \tag{B11}$$

Now require:

$$\frac{\partial \ln L}{\partial \lambda_o} = \frac{n}{\lambda_o} - \sum_{j=1}^{k} a^j \sum_{i \varepsilon I_j} x_i = 0 \tag{B12}$$

$$\frac{\partial \ln L}{\partial a} = \frac{1}{a} \sum_{j=1}^{k} j\, n_j - \lambda_o \sum_{j=1}^{n} j\, a^{j-1} \sum_{i \varepsilon I_j} x_i = 0 \tag{B13}$$

Combining (B12) and (B13) gives the estimate for a as the solution of

$$\frac{n \sum_{j=1}^{k} j\, a^j \sum_{i \varepsilon I_j} x_i}{\sum_{j=1}^{k} j\, n_j} - \sum_{j=1}^{k} a^j \sum_{i \varepsilon I_j} x_i = 0 \tag{B14}$$

and the estimate of λ_o is found from (B12)

$$\lambda_o = \frac{n}{\sum_{j=1}^{k} a^j \sum_{i \varepsilon I_j} x_i} \tag{B15}$$

For model II the hazard function is

$$z_j = \lambda_o\, a^{M_j} \tag{B16}$$

where

$$M_j = n_1 + n_2 + \ldots n_{j-1} \tag{B17}$$

The likelihood here is

$$L(x_1 . . x_n) = \prod_{i \varepsilon I_1} \lambda_o\, e^{-\lambda_o x_i} \ldots \prod_{i \varepsilon I_k} \lambda_o\, a^{M_k}\, e^{-\lambda_o a^{M_k} x_i} \tag{B18}$$

and

$$\ln L\ (x_1\ .\ .\ x_n) = \sum_{j=1}^{k} \sum_{i\varepsilon I_j} (\ln\lambda_o + M_j \ln a - \lambda_o a^{M_j} x_i)$$

$$= n \ln \lambda_o + \ln a \sum_{j=1}^{k} n_j M_j - \lambda_o \sum_{j=1}^{k} a^{M_j} \sum_{i\varepsilon I_j} x_i \qquad \text{(B19)}$$

Require

$$\frac{\partial \ln L}{\partial \lambda_o} = \frac{n}{\lambda_o} - \sum_{j=1}^{k} a^{M_j} \sum_{i\varepsilon I_j} x_i = 0 \qquad \text{(B20)}$$

and

$$\frac{\partial \ln L}{\partial a} = \frac{1}{a} \sum_{j=1}^{k} n_j M_j - \lambda_o \sum_{j=1}^{k} M_j\ a^{M_j - 1} \sum_{i\varepsilon I_j} X_i = 0 \qquad \text{(B21)}$$

Combine (B20) with (B21) to form:

$$\frac{n \sum_{j=1}^{k} (M_j\ a^{M_j} \sum_{i\varepsilon I_j} x_i)}{\sum_{j=1}^{k} n_j\ M_j} - \sum_{j=1}^{k} a^{M_j} \sum_{i\varepsilon I_j} x_i = 0 \qquad \text{(B22)}$$

Eq. (B22) is solved for $\hat{a}$. Later, $\hat{\lambda}_o$ is found from (B20)

$$\lambda_o = \frac{n}{\sum_{j=1}^{k} (a^{M_j} \sum_{i\varepsilon I_j} x_i)} \qquad \text{(B23)}$$

B.3 Least Square x Model

The estimation error to be minimized is

$$E = \sum_{i=1}^{n} (x_i - \bar{x}_i)^2 = \sum_{j=1}^{k} \sum_{i \varepsilon I_j} (x_i - \frac{1}{N_j \phi})^2 \qquad \text{(B24)}$$

Note that $N_j = N - n_1 - \dots - n_{j-1}$ (B25)

Therefore $\frac{\partial N_j}{\partial N} = 1$ (B26)

Now require

$$\frac{\partial E}{\partial N} = 2 \sum_{j=1}^{k} \sum_{i \varepsilon I_j} [(x_i - \frac{1}{\phi N_j}) \cdot (\frac{1}{\phi N_j^2})] = 0 \qquad \text{(B27)}$$

and

$$\frac{\partial E}{\partial \phi} = 2 \sum_{j=1}^{k} \sum_{i \varepsilon I_j} [(x_i - \frac{1}{\phi N_j}) (\frac{1}{\phi^2 N_j})] = 0 \qquad \text{(B28)}$$

Rewrite (B27) and (B28) as

$$\phi \sum_{j=1}^{k} (\frac{1}{N_j^2} \sum_{i \varepsilon I_j} x_i) - \sum_{j=1}^{k} \frac{n_j}{N_j^3} = 0 \qquad \text{(B29)}$$

$$\phi \sum_{j=1}^{k} (\frac{1}{N_j} \sum_{i \varepsilon I_j} x_i) - \sum_{j=1}^{k} \frac{n_j}{N_j^2} = 0 \qquad \text{(B30)}$$

Now, combine (B-29) and (B-30) to form:

$$\frac{\sum_{j=1}^{k} \frac{n_j}{N_j^3} - \sum_{j=1}^{k} \left(\frac{1}{N_j^2} \sum_{i\epsilon I_j} x_i\right)}{\sum_{j=1}^{k} \frac{n_j}{N_j^2} - \sum_{j=1}^{k} \left(\frac{1}{N_j} \sum_{i\epsilon I_j} x_i\right)} = 0 \tag{B31}$$

Solve (B31) for N and then evaluate ϕ from (B30)

$$\phi = \frac{\sum_{j=1}^{k} \frac{n_j}{N_j^2}}{\sum_{i=1}^{k} \left(\frac{1}{N_j} \sum_{i\epsilon I_j} x_i\right)} \tag{B32}$$

B.4 Least Square t Model

The estimation error is

$$E = \sum_{i=1}^{n} \left(t_i - \sum_{m=1}^{i} \frac{1}{\phi N_{(m)}}\right)^2 \tag{B33}$$

In order to minimize E require

$$\frac{\partial E}{\partial N} = \sum_{i=1}^{n} \left[2\left(t_i - \sum_{m=1}^{i} \frac{1}{\phi N_{(m)}}\right) \left(\sum_{m=1}^{i} \frac{1}{\phi N^2_{(m)}}\right)\right] = 0 \tag{B34}$$

Also

$$\frac{\partial E}{\partial \phi} = \sum_{i=1}^{n} \left[2\left(t_i - \sum_{m=1}^{i} \frac{1}{\phi N_{(m)}}\right)\left(\sum_{m=1}^{i} \frac{1}{\phi^2 N_{(m)}}\right)\right] = 0 \tag{B35}$$

Rewrite (B34) and (B35) as

$$\phi \sum_{i=1}^{n} \left(t_i \sum_{m=i}^{i} \frac{1}{N^2_{(m)}}\right) - \sum_{i=1}^{n} \left(\sum_{m=1}^{i} \frac{1}{N_{(m)}} \sum_{m=1}^{i} \frac{1}{N^2_{(m)}}\right) = 0 \tag{B36}$$

$$\phi \sum_{i=1}^{m} (t_i \sum_{m=1}^{i} \frac{1}{N_{(m)}}) - \sum_{i=1}^{n} (\sum_{m=1}^{i} \frac{1}{N_{(m)}})^2 = 0 \tag{B37}$$

(B-36) and (B-37) can be simplified by using the notation

$$A_i = \sum_{m=1}^{i} \frac{1}{N_{(m)}} \tag{B38}$$

$$B_1 = \sum_{m=1}^{i} \frac{1}{N^2_{(m)}} \tag{B39}$$

This allows writing (B36) and (B37) as

$$\phi \sum_{i=1}^{n} t_i B_i - \sum_{i=1}^{n} A_i B_i = 0. \tag{B40}$$

$$\phi \sum_{i=1}^{n} t_i A_i - \sum_{i=1}^{n} A_i^2 = 0. \tag{B41}$$

(B40) and (B41) may be combined to form

$$\sum_{i=1}^{n} t_i B_i \sum_{i=1}^{n} A_i^2 - \sum_{i=1}^{n} t_i A_i \sum_{i=1}^{n} A_i B_i = 0 \tag{B42}$$

and

$$\phi = \frac{\sum_{i=1}^{n} A_i^2}{\sum_{i=1}^{n} t_i A_i} \tag{B43}$$

Note that (B42) and (B43) are identical to (A32) and (A33), except that A_i and B_i are defined slightly differently.

B.5 Geometric Least Square x Model

The estimation error here is

$$E = \sum_{i=1}^{n} (x_i - \bar{x}_i)^2 = \sum_{i=1}^{n} \left(x_i - \frac{1}{\lambda_o\, a^{M(i)}}\right)^2 \tag{B44}$$

To minimize E, require:

$$\frac{\partial E}{\partial a} = \sum_{i=1}^{n} 2\left[\left(x_i - \frac{1}{\lambda_o\, a^{M(i)}}\right)\left(\frac{M_{(i)}}{\lambda_o\, a^{M(i)+1}}\right)\right] = 0 \tag{B45}$$

$$\frac{\partial E}{\partial \lambda_o} = \sum_{i=1}^{n} 2\left[\left(x_i - \frac{1}{\lambda_o\, a^{M(i)}}\right)\left(\frac{1}{\lambda_o^2\, a^{M(i)}}\right)\right] = 0 \tag{B46}$$

Rewrite the above as

$$\lambda_o \sum_{i=1}^{n} \frac{x_i\, M_{(i)}}{a^{M(i)}} - \sum_{i=1}^{n} \frac{M_{(i)}}{a^{2M}{(i)}} = 0 \tag{B47}$$

$$\lambda_o \sum_{i=1}^{n} \frac{X_i}{a^{M(i)}} - \sum_{i=1}^{n} \frac{1}{a^{2M(i)}} = 0 \tag{B48}$$

Combine (B47) and (B48) to form

$$\sum_{i=1}^{n} \frac{x_i\, M_{(i)}}{a^{M}{(i)}} \quad \sum_{i=1}^{n} \frac{1}{a^{2M}{(i)}} \quad \sum_{i=1}^{n} \frac{x_i}{a^{M}{(i)}} \quad \sum_{i=1}^{n} \frac{M_{(i)}}{a^{2M(i)}} = 0 \tag{B49}$$

The solution of (B49) gives $\hat{a}$, the best estimate of a. λ_o is found from (B48) to be

$$\lambda_o = \frac{\sum_{i=1}^{n} \frac{1}{a^{2M(i)}}}{\sum_{i=1}^{n} \frac{x_i}{a^{M(i)}}} \tag{B50}$$

B.6 Geometric Least Square t Model

The estimation error to be minimized is

$$E = \sum_{i=1}^{n} \left(t_i - \sum_{m=1}^{i} \frac{1}{\lambda_o a^{M(m)}}\right)^2 \tag{B51}$$

Require:

$$\frac{\partial E}{\partial a} = \sum_{i=1}^{n} \left[2\left(t_i - \sum_{m=1}^{i} \frac{1}{\lambda_o a^{M(m)}}\right)\left(\sum_{m=1}^{i} \frac{M_{(m)}}{\lambda_o a^{M_{(m)}+1}}\right)\right] = 0 \tag{B52}$$

and

$$\frac{\partial E}{\partial \lambda_o} = \sum_{i=1}^{n} \left[2\left(t_i - \sum_{m=1}^{i} \frac{1}{\lambda_o a^{M(m)}}\right)\left(\sum_{m=1}^{i} \frac{1}{\lambda_o^2 a^{M(m)}}\right)\right] = 0 \tag{B53}$$

Rewrite the above equations as:

$$\lambda_o \sum_{i=1}^{n} \left(t_i \sum_{m=1}^{i} \frac{M_{(m)}}{a^{M(m)}}\right) - \sum_{i=1}^{n} \left(\sum_{m=1}^{i} \frac{1}{a^{M(m)}} \cdot \sum_{m=1}^{i} \frac{M_{(m)}}{a^{M(m)}}\right) = 0 \tag{B54}$$

$$\lambda_o \sum_{i=1}^{n} \left(t_i \sum_{m=1}^{i} \frac{1}{a^{M(m)}}\right) - \sum_{i=1}^{n} \left(\sum_{m=1}^{i} \frac{1}{a^{M(m)}}\right)^2 = 0 \tag{B55}$$

Define

$$A_i = \sum_{m=1}^{i} \frac{1}{a^{M(m)}} \tag{B56}$$

and

$$B_i = \sum_{m=1}^{i} \frac{M(m)}{a^{M(m)}} \tag{B57}$$

Then we can rewrite (B54) and (B55) as

$$\lambda_o \sum_{i=1}^{n} t_i B_i - \sum_{i=1}^{n} A_i B_i = 0 \tag{B58}$$

$$\lambda_o \sum_{i=1}^{n} t_i A_i - \sum_{i=1}^{n} A_i^2 = 0 \tag{B59}$$

Then $\hat{a}$ can be found from:

$$\sum_{i=1}^{n} t_i B_i \sum_{i=1}^{n} A_i^2 - \sum_{i=1}^{n} t_i A_i \sum_{i=1}^{n} A_i B_i = 0 \tag{B60}$$

and $\hat{\lambda}_o$ is found from

$$\lambda_o = \frac{\sum_{i=1}^{n} A_i^2}{\sum_{i=1}^{n} t_i A_i} \tag{B61}$$

Appendix C--Derivation of $Var(\hat{N})$ and $Var(\hat{T})$

Let the variables $x_1, x_2, \ldots x_n$ have a probability density function.

$$f(x_1, x_2, \ldots x_n; \theta_1, \theta_2, \ldots \theta_n)$$

If $\hat{\theta}_1, \hat{\theta}_2, \ldots \hat{\theta}_n$, are the maximum likelihood estimators of $\theta_1, \theta_2, \ldots \theta_n$, and n is large, then $\theta_1, \theta_2, \ldots \theta_n$ are approximately distributed by the multivariate normal distribution with means $\theta_1, \theta_2, \ldots \theta_n$. Moreover, if we define the matrix R to have the elements

$$r_{ij} = - E[\frac{\partial^2}{\partial\theta_i \partial\theta_j} \ln f(x_1, x_2, \ldots x_n; \theta_1, \theta_2, \ldots \theta_n)] \quad \text{(c-1)}$$

then the variance matrix, V, equals

$$V = R^{-1} \quad \text{(c-2)}$$

Next, we use (c-1) and (c-2) to derive $Var(\hat{N})$ and $Var(\hat{T})$.

C.1. Derivation of $Var(\hat{N})$

Consider the probability function

$$f(x_1, x_2, \ldots x_n; N,\phi) = \prod_{i=1}^{n} \phi(N-i+1)\, e^{-\phi(N-i+1)x_i} \quad \text{(c-3)}$$

and

$$\ln f(x_1, x_2, \ldots x_n; N,\phi) = n \ln \phi + \sum_{i=1}^{n} \ln(N-i+1) - \phi \sum_{i=1}^{n} (N-i-1)x_i \quad \text{(c-4)}$$

Next, differentiate (c-4) in order to obtain the r_{ij} terms

$$\frac{\partial \ln f}{\partial \phi} = \frac{n}{\phi} - \sum_{i=1}^{n} (N-i+1)\, x_i$$

$$\frac{\partial^2 \ln f}{\partial \phi^2} = \frac{-n}{\phi^2}$$

$$\frac{\partial^2 \ln f}{\partial \phi, \partial N} = - \sum_{i=1}^{n} x_i$$

$$\frac{\partial \ln f}{\partial N} = \sum_{i=1}^{n} \frac{1}{(N-i+1)} - \phi \sum_{i=1}^{n} x_i$$

$$\frac{\partial^2 \ln f}{\partial N^2} = \sum_{i=1}^{n} \frac{-1}{(N-i+1)^2}$$

Define the quantities A and S by

$$A = \sum_{i=1}^{n} x_i \qquad \text{(c-5)}$$

$$S = \sum_{i=1}^{n} \frac{1}{(N-i+1)^2} \qquad \text{(c-6)}$$

Then we can express R by

$$R = \begin{bmatrix} S & A \\ A & \frac{n}{\phi^2} \end{bmatrix} \qquad \text{(c-7)}$$

In order to find the inverse of R, note that the determinant, D, is

$$D = \frac{Sn}{\phi^2} - A^2 \qquad \text{(c-8)}$$

Then the inverse matrix is

$$V = R^{-1} = \begin{bmatrix} \frac{n}{\phi^2 D} & \frac{-A}{D} \\ \frac{-A}{D} & \frac{S}{D} \end{bmatrix} \quad \text{(c-9)}$$

We are interested in $Var(\hat{N})$ which equals

$$Var(\hat{N}) = \frac{n}{\phi^2 D} = \frac{n}{nS - A^2\phi^2} \quad \text{(c-10)}$$

C.2 Derivation of $Var(\hat{T})$

We wish to express the probability density function of $x_1, x_2, \ldots x_n$, in terms of N and T, where T is the mean time to failure after the correction of n errors. This is done by noting that

$$T = \frac{1}{\phi(N-n)} \quad \text{(c-11)}$$

Then we may write the probability density function as

$$f(x_1, x_2, \ldots x_n; N, T) = \prod_{i=1}^{n} \frac{(N-i+1)}{(N-n)\, T}\, e^{-\frac{(N-i+1)\, x_i}{(N-n)\, T}} \quad \text{(c-12)}$$

Therefore,

$$\ln f(x_1, x_2 \ldots x_n; N,T) = \Sigma \ln (N-i+1) - n \ln (N-n) - n \ln T$$

$$- \frac{1}{T} \sum_{i=1}^{n} x_i - \frac{1}{(N-n)T} \sum_{i=1}^{n} (n-i+1)\, x_i \quad \text{(c-12)}$$

Recall (c-5) and (c-6) and define

$$B = \sum_{i=1}^{n} (n-i+1)\, x_i \tag{c-13}$$

Then we may write $\ln f(x_1, x_2, \ldots x_n; N,T)$ as

$$\ln f(x_1, x_2, \ldots x_n; N,T) = \sum_{i=1}^{n} \ln(N-i+1) - n \ln (N-n) - n \ln T$$

$$- \frac{A}{T} - \frac{B}{(N-n)T} \tag{c-14}$$

The partial derivatives of $\ln f(\cdot)$ are:

$$\frac{\partial^2 \ln f}{\partial N^2} = \sum_{i=1}^{n} \frac{-1}{(N-i+1)^2} + \frac{n}{(N-n)^2} - \frac{2B}{(N-n)^3\, T}$$

$$\frac{\partial^2 \ln f}{\partial N, \partial T} = \frac{-N}{(N-n)^2\, T^2}$$

$$\frac{\partial^2 \ln f}{\partial T^2} = \frac{n}{T^2} - \frac{2A}{T^3} - \frac{2B}{(N-n)\, T^3}$$

The matrix R is

$$R = \begin{bmatrix} S - \dfrac{n}{(N-n)^2} + \dfrac{2B}{(N-n)^3\, T} & \dfrac{B}{(N-n)^2\, T^2} \\ \\ \dfrac{B}{(N-n)^2\, T^2} & - \dfrac{n}{T^2} + \dfrac{2A}{T^3} + \dfrac{2B}{(N-n)\, T^3} \end{bmatrix} \tag{c-15}$$

The determinant of R is

$$\Delta = \left(S - \frac{n}{(N-n)^2} + \frac{2B}{(N-n)^3 T}\right)\left(\frac{-n}{T^2} + \frac{2A}{T^3} + \frac{2B}{(N-n)\ T^3}\right) - \frac{B^2}{(N-n)^4\ T^4} \qquad \text{(c-16)}$$

and the variance matrix, V, is

$$V = R^{-1} = \frac{1}{\Delta} \begin{bmatrix} \frac{-n}{T^2} + \frac{2A}{T^3} + \frac{2B}{(N-n)\ T^3} & \frac{-B}{(N-n)^2\ T^2} \\ \frac{-N}{(N-n)^2\ T^2} & S - \frac{n}{N-n} + \frac{2B}{(N-n)^3\ T} \end{bmatrix}$$

The variance of $\hat{T}$ is therefore:

$$\mathrm{Var}(\hat{T}) = \frac{1}{\Delta}\left(S - \frac{n}{(N-n)^2} + \frac{2B}{(N-n)^3\ T}\right) \qquad \text{(c-18)}$$

The expressions (c-10) and (c-18) are used to determine the confidence intervals in Section 6.

Appendix D - A General Method for Obtaining Confidence Intervals

The method described in this section does not rely on the assumption that the sample size is large. Therefore, it is applicable to other cases as well.

Suppose that we have detected n errors with the inter-arrival times $x_1, x_2, \ldots x_n$. On the basis of those we estimate the initial number of errors in the program $\hat{N}(x_1, x_2, \ldots x_n)$. Suppose at this point that we can determine the probability density of $\hat{N}$ as a function of the true value, $g(\hat{N};N)$. This point will be discussed later. Suppose, for definiteness that we need a 90 percent confidence level. If any arbitrary number, say N', is substituted for N in $g(\hat{N};N)$, the distribution of $\hat{N}$ will be completely specified and it will be possible to make statements about $\hat{N}$. In particular, we may find two numbers L and H such that

$$P(\hat{N}<L) = \int_n^L g(\hat{N};N)\, d\hat{N} = 0.05 \tag{D-1}$$

$$P(\hat{N}>H) = \int_H^\infty g(\hat{N};N)\, d\hat{N} = 0.05 \tag{D-2}$$

The numbers L and H will depend, of course, on the number substituted for N in $g(\hat{N};N)$. In fact, we may write L and H as functions of N; L(N) and H(N). The values of L and H for any value of N are determined by equations (D-1) and (D-2). Clearly

$$P[L(N) < \hat{N} < H(N)] = \int_L^H g(\hat{N};N)\, dN = 0.9 \tag{D-3}$$

L(N) and H(N) may be plotted against N as in Fig. D.1. A vertical line

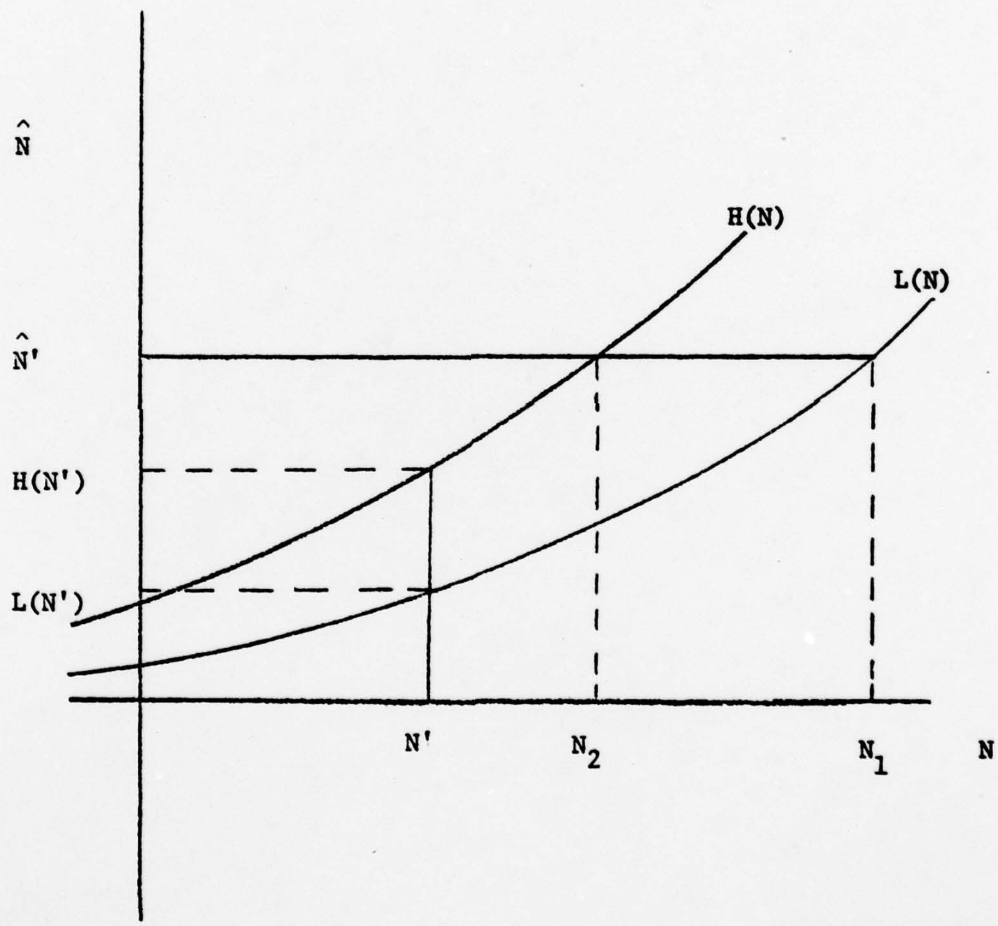

Fig. D.1 Graphical interpretation of the method for obtaining confidence intervals

through any chosen value of N' will intersect the two curves in points which, projected on the $\hat{N}$ axis, will give limits between which $\hat{N}$ will fall with probability 0.90.

Having constructed the two curves $\hat{N} = L(N)$ and $\hat{N} = H(N)$, we may construct a confidence interval for N as follows: On the basis of the sample of n failures compute the value of the estimator, say $\hat{N}'$. A horizontal line through the point $\hat{N}'$ on the $\hat{N}$ axis (Fig. D.1) will intersect the two curves at points which may be projected on the N axis and labeled N_1 and N_2, as in the figure. These two numbers define the confidence interval, for it is easily shown that

$$P(N_1 < N < N_2) = 0.90 \tag{D-4}$$

In order to clarify this point suppose that the number of error is N'. The probability that the estimate will fall between L(N') and H(N') is 0.90. If the estimate does fall between these limits, then the horizontal line will cut the veritcal line, which goes through N', at some point between the curves, and the corresponding interval $(N_2\ N_1)$ will cover N'. If the estimate does not fall between L(N') and H(N'), the horizontal line does not cut the vertical line between the curves, and the corresponding interval (N_2, N_1) does not cover N'. It follows, therefore, that the probability is exactly 0.90 that an interval (N_2, N_1) constructed by this method will cover N'. This is true for any value of N.

It is possible to determine the limits N_2 and N_1 for a given estimate without finding the curves L(N) and H(N). Referring to Fig. D.1, the limits for N are the points N_2 and N_1 such that

$L(N_1) = \hat{N}'$ and $H(N_2) = \hat{N}'$. Thus, instead of finding the two curves, we may solve for the points N_1 and N_2 which satisfy these conditions.

In order to apply this method we have to determine the probability density of the estimator $g(\hat{N};N)$. Furthermore, we have to show first that $g(\hat{N};N)$ depends only on N. We may start with the general assumption that $\hat{N}$ depends on all the system parameters, that is, $g(\hat{N};N,\phi,n)$. Since n, the number of corrected errors is known, n is a known quantity and not a parameter. Next, we have to show that $\hat{N}$ is independent of ϕ, in order to reduce $g(\hat{N};N,\phi)$ to the desired form.

Suppose that instead of estimating $\hat{N}$ from $x_1, x_2 \ldots x_n$, we estimate it from a new sequence, $y_1, y_2, \ldots y_n$, defined as

$$y_i = \phi x_i \qquad \text{(D-5)}$$

Since the probability density function of x_i is

$$f(x_i) = (N-i+1)\phi \; e^{-(N-i+1)\phi \, x_i} \qquad \text{(D-6)}$$

The probability density function of y_i can be found from (D-7)

$$f_Y(y_i) = \left|\frac{dx}{dy}\right| \; f_X\,(x_i) \qquad \text{(D-7)}$$

This is found to be

$$f(y_i) = (N-i+1) \; e^{-(N-i+1)\, y_i} \qquad \text{(D-8)}$$

Thus, the new random variable, y_i, is normalized such that it is independent of ϕ. If we estimate N on the basis of the y_i sequence, the resulting estimate $\hat{N}$ will be independent of ϕ. Recall Eq. (A.7) which is used to estimate $\hat{N}$.

$$\sum_{i=1}^{n} \frac{1}{\hat{N}-i+1} = \frac{n}{\hat{N} - \frac{\sum_{i=1}^{n} (i-1)\, x_i}{\sum_{i=1}^{n} x_i}} \tag{D-9}$$

Now, rewrite (D-9) as

$$\sum_{i=1}^{n} \frac{1}{\hat{N}-i+1} = \frac{n}{\hat{N} - \frac{\sum_{i=1}^{n} (i-1)\, y_i}{\sum_{i=1}^{n} y_i}} \tag{D-10}$$

Note that (D-10) describes $\hat{N}$ as a function of the y's, and therefore, $\hat{N}$ is independent of ϕ. Thus, we have established that $\hat{N}$ is a function of the parameter N and the known quantity n. Therefore, we may write the density of $\hat{N}$ as $g(\hat{N};N)$. In order to construct $g(\hat{N};N)$, it is realized that an analytical derivation is impossible, and therefore, a simulation is used to find $g(\hat{N};N)$. This is done by generating 1000 sequences of x_i variables with the desired probability density function with N being set to some fixed value N', and n is given. For each sequence we evaluate $\hat{N}$, and we end up with 1000 estimates of $\hat{N}$. The histogram of $\hat{N}$ is a numerical approximation for $g(N;N')$. Note that this only gives $g(\hat{N},N')$ for one point, N=N'. However, we need to find $g(\hat{N};N)$ for only a few points, as explained above.

The histograms were generated for various values of ϕ, and it was found, as expected from theory, that the histograms, and $g(\hat{N};N)$ are independent of ϕ.

The method described above can be modified easily to the case where a one-sided interval is needed. In that case we only have to construct L(N) and determine from it N_1. The confidence interval for this case is $n<N<N_1$.

This method can be extended to the case where the estimator is a function of two parameters, such as for $\hat{T}$, $g(\hat{T}; T,N)$. However, the construction of the histograms with two parameters becomes very complex and therefore this approach is abandoned. On the other hand, it is impossible to express $\hat{T}$ as a function of a single parameter, $g(\hat{T};T)$. This made the present method attractive only for $\hat{N}$, whereas the confidence intervals for $\hat{T}$ are determined by the method of Section 6.1

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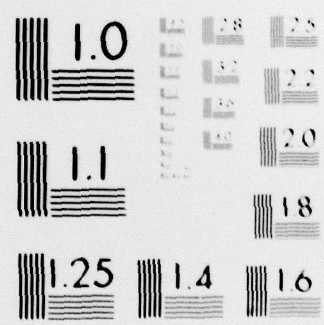

MICROCOPY RESOLUTION TEST CHART
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Appendix E--Error Data Collection Format

Three separate tables are needed for adequate cross correlation and configuration control of the error data.

Table #1 will provide information concerning the test runs (where run is defined as being the execution or attempted execution of a specific test case). It is assumed that the entire OFP will be in residence in the FCC computer during the execution of each test run. It is also assumed that all test runs (including those which discovered no errors) will be listed here. This is critical as the model will be attempting to calculate an MTBF. The format is as follows:

Run #	Calendar Date of Run	Time of Day of Run	OFP Type Configuration #	Run Length (Sec)	Short description of test case. E.G., Bus control component, verify transmission word count.

Table #2 will provide tape configuration information. The format for this table follows:

Tape Configuration #	Calendar Date it Replaced Previous Tape	List of Changes from Previous Tape Configuration

Table #3 provides the information on software errors discovered during the testing. This table requests the execution time at which an error occurred. This means that recorded data used to search for an error will have to be time correlated to the FCC execution (preferably to within a major cycle). This is intended to be the execution time for the error source rather than the error symptom. For example, if an incorrect display is discovered, we want to know the execution time at which the parameter being displayed was incorrectly calculated (or output or formatted etc.) rather than the time at which the faulty display was noticed. This table also requests that the source component be identified.. Here again we are not interested in symptoms. If a single symptom is caused by several sources, each source should be listed as a separate error. It should also be noted that a single source may cause several symptoms. In this case we are interested in only the source. Thus the errors recorded in this table are not necessarily synonymous with anomaly reports. The above requests will require considerable analysis of discovered anomalies. If this analysis is out of your scope, please so inform us. The format for Table #3 is as follows:

Error#	Run #[1]	Execution time of Error Occurrance	Source Component of Error	Tape Configuration No.	Anomaly Report #[2]	Error Cat[3]

NOTES:

1. This is the run # in error was first discovered.

2. This refers to anomaly reports for which this error is a source (may be more than a single report).

3. One of the following categories:

a. Computational; e.g., index, equation, sign convention, modeling, mixed mode, truncation, rounding, units, convergence, etc.

b. Logical; e.g., limit determination, logic branch, loop exit, missing condition, flag, iteration step size, storage reference, endless loop, etc.

c. I/O; e.g., missing I/O, garbed I/O, wrong field size, format, control, discrete usage, etc.

d. Data handling; e.g., data lost, write or read to wrong location, number of entries, index or flag modification, bit manipulation, number type conversion, subscripting, bounds, etc.

e. Configuration; e.g., compilation, segmentation, illegal instruction, etc.

f. Routine/routine interface; e.g., pass wrong parameters, expect wrong parameters, communicate with wrong data block, calling sequence, etc.

g. User interface; e.g., data read but not used, data rejected but used, valid data rejected, incorrect mode change, etc.

h. Data base; e.g., uncoordinated use of data elements, incorrect initialization, missing data, wrong location, etc.

i. Requirements compliance; e.g., duty cycle violated, specified accuracy not met, specified timing not met.

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